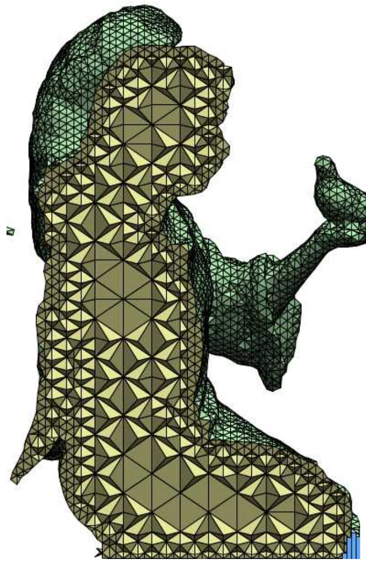
Lautsch Finite Elemente GmbH

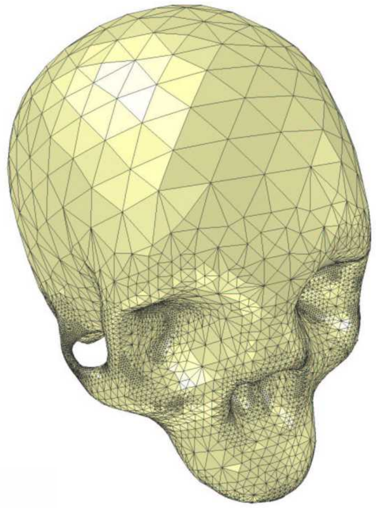
The Multigrid Marching Tetra Method

Summary
From 2D to 3D
Gallery
Outlook

To refine a BCC mesh is the best way of adaptive 3D approximation.

We follow the work of Molino (2002)



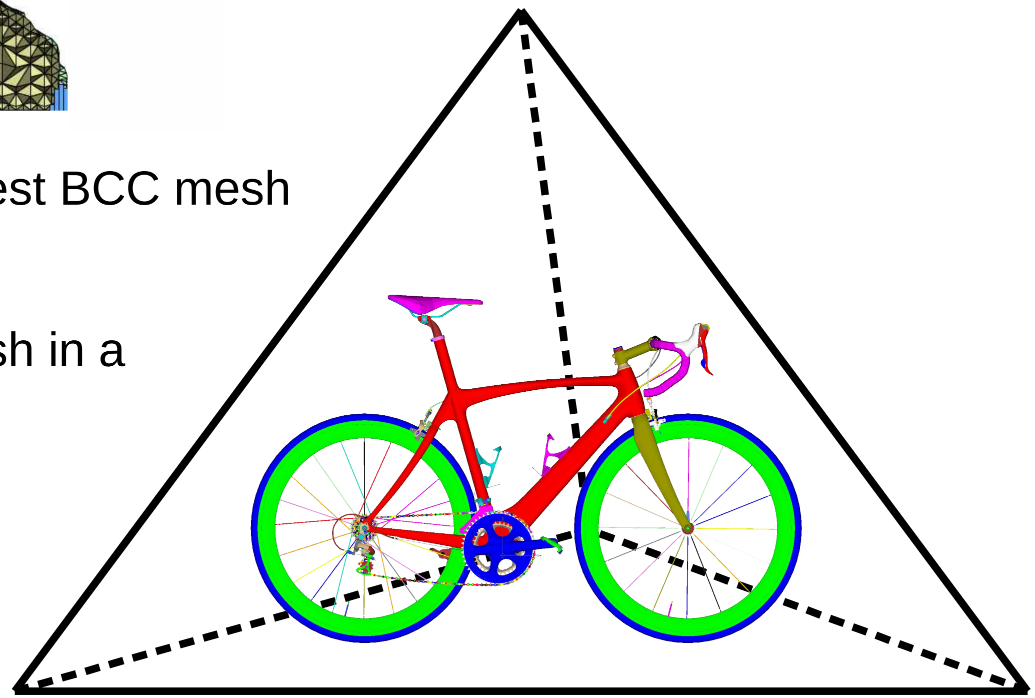


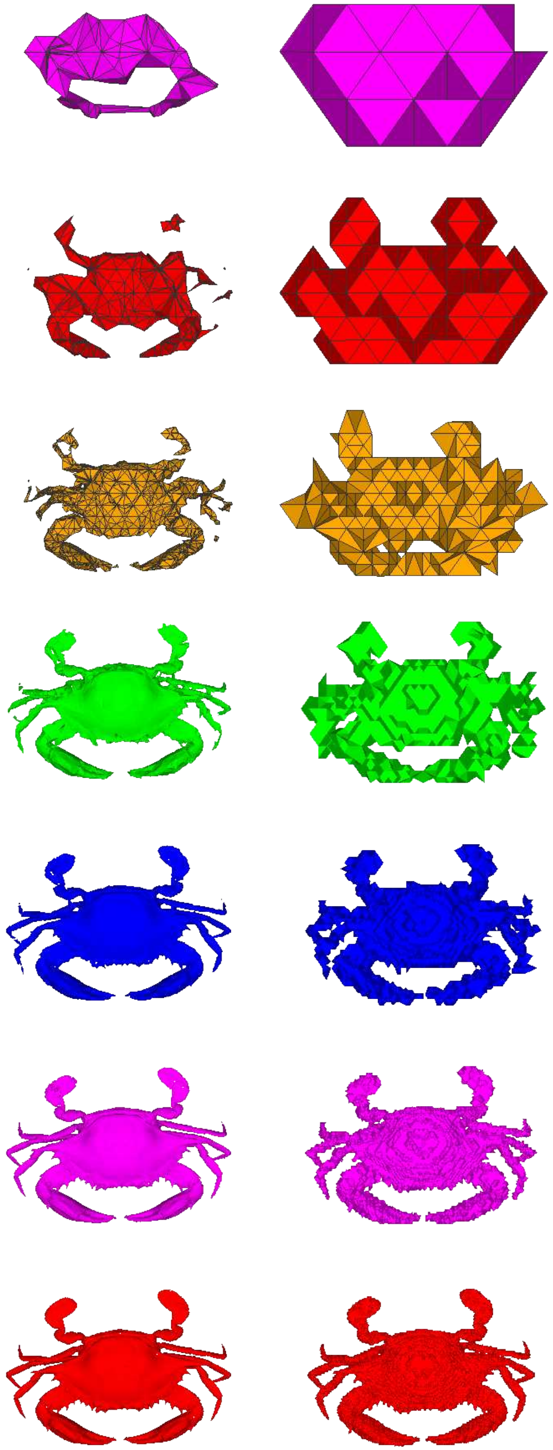
and Chewschuk (2007)

and many others.

In contrast to them we start with the coarsest BCC mesh
that ever is possible - a single tetra.

And we design the transition to the FE mesh in a
different manner which allows to consider
sharp edges and 10 000 parts.





The MMT mesh is a refinement of an adapted BCC mesh.

The name of the method is „Multigrid Marching Tetra“

Multigrid	we are working with a hierarchical sequence of meshes, similar to the meshes that were used from 1980 to 2000 to speed up the iterative solution of large sytems of linear equations. To solve these systems is a numerical bottle neck of FEA since it requires more than O(n) operations.
Marching Tetra	Today dfferent methods are used and each of them has its own adaptation rules.
Goal	It would be great to have a method which after some iterations says „no more refinement is needed, since the approximation of the input geometry is exact.“

… at least for simple parts.

The same holds for the requirements of adaptive FEA.

Adaptive BCC refinement

Tetras may belong to different parts

Each node belongs to one part

Last refinement : Marching Tetra

Each tetra belongs to one part

nodes may belong to different parts
e.g. surface nodes

Marching Tetra introduces the part geometry to the locally refined BCC mesh.

Marching Tetra creates bad elements, but this is not a problem:

Good elements of the BCC mesh approximate well.

Better elements can be achieved in a final step.

End of the summary.

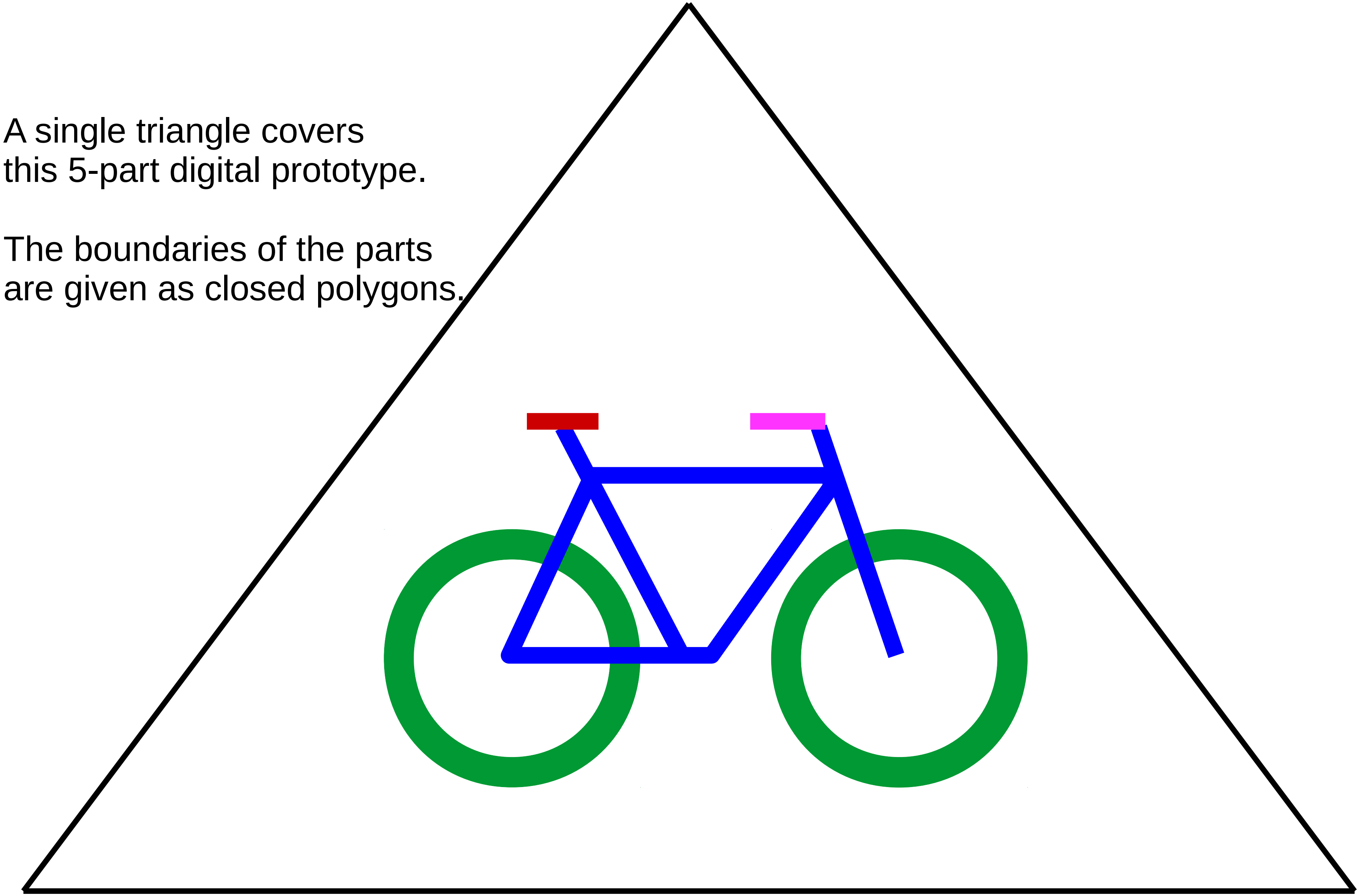
From 2D to 3D

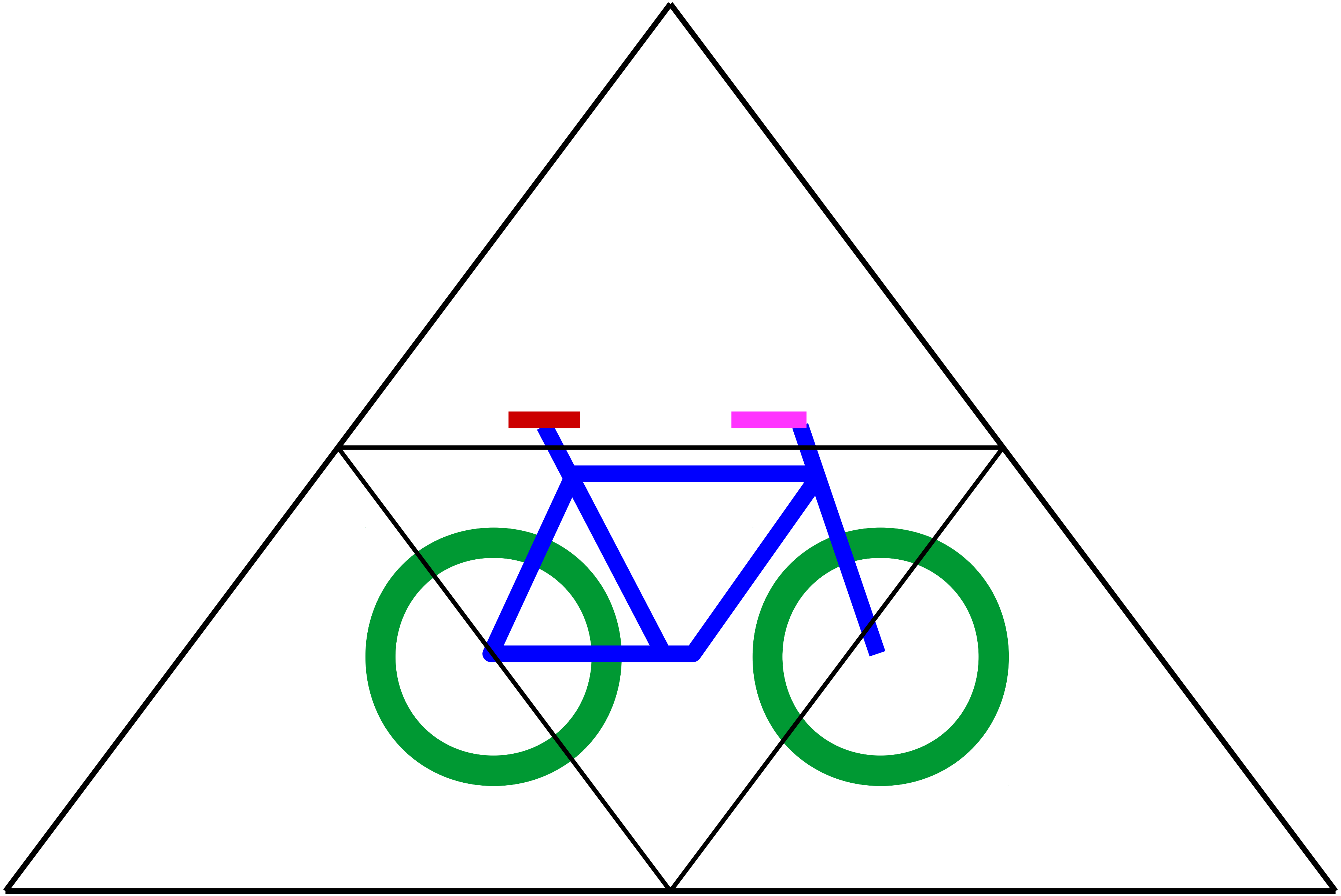
We have a procedure which for any 2 points A, B of R^2 or R^3 determines: The straight line from A to B crosses the boundaries of parts N, NN, … NNN. Location of the crossing and normal of the part are stored.

We discuss a 2D version and explain the extensions to the 3D case.

A single triangle covers
this 5-part digital prototype.

The boundaries of the parts
are given as closed polygons.

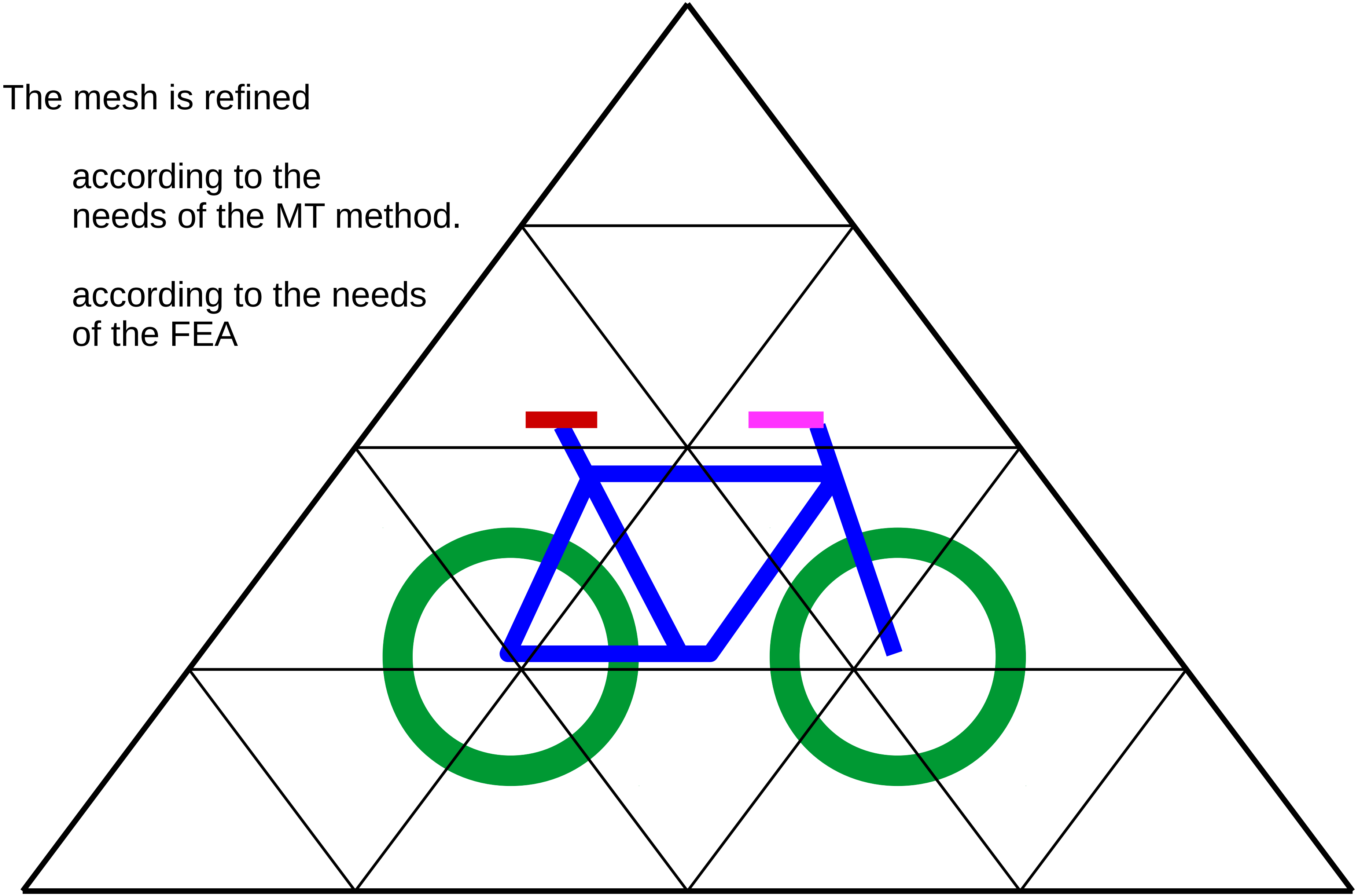


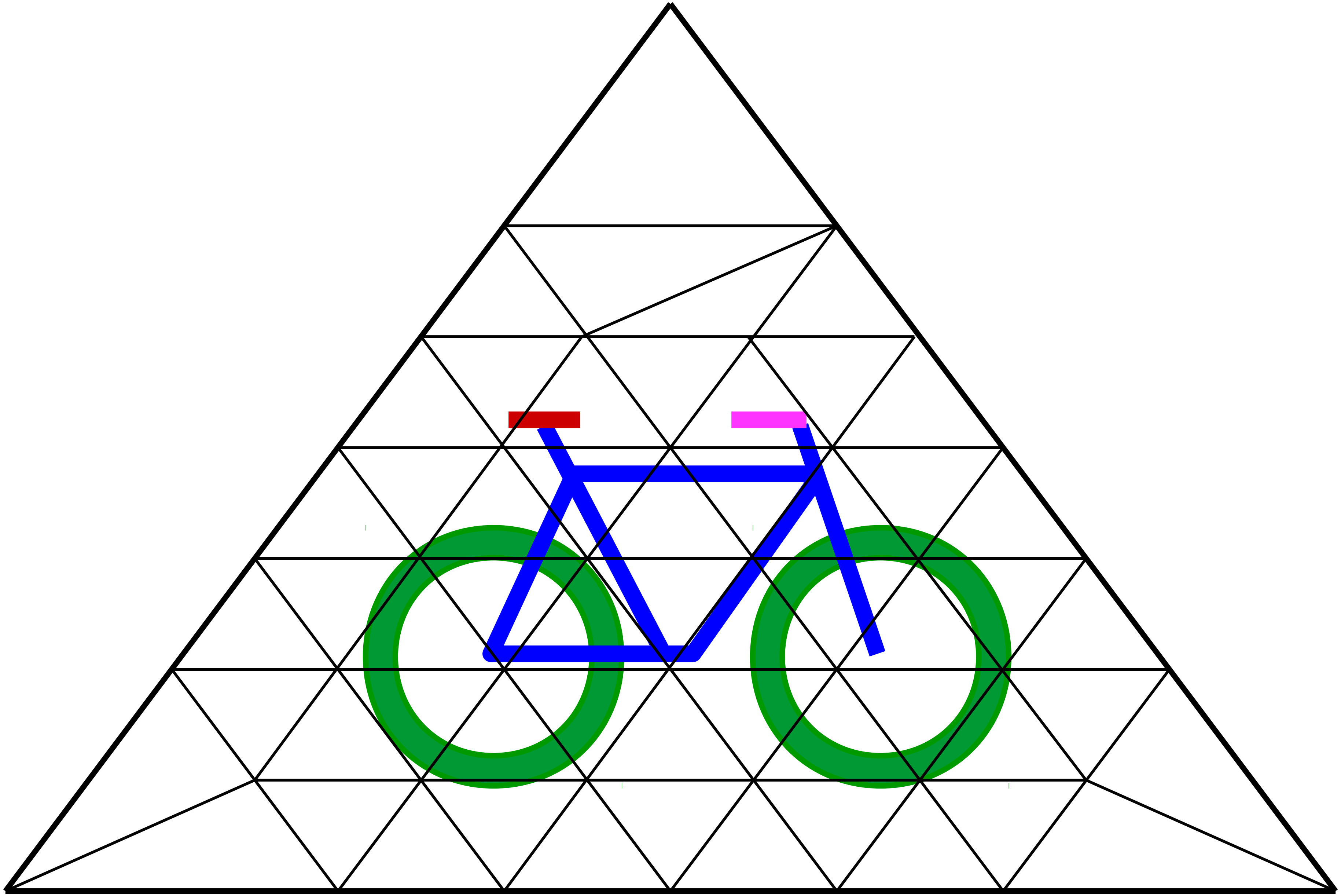


The mesh is refined

according to the
needs of the MT method.

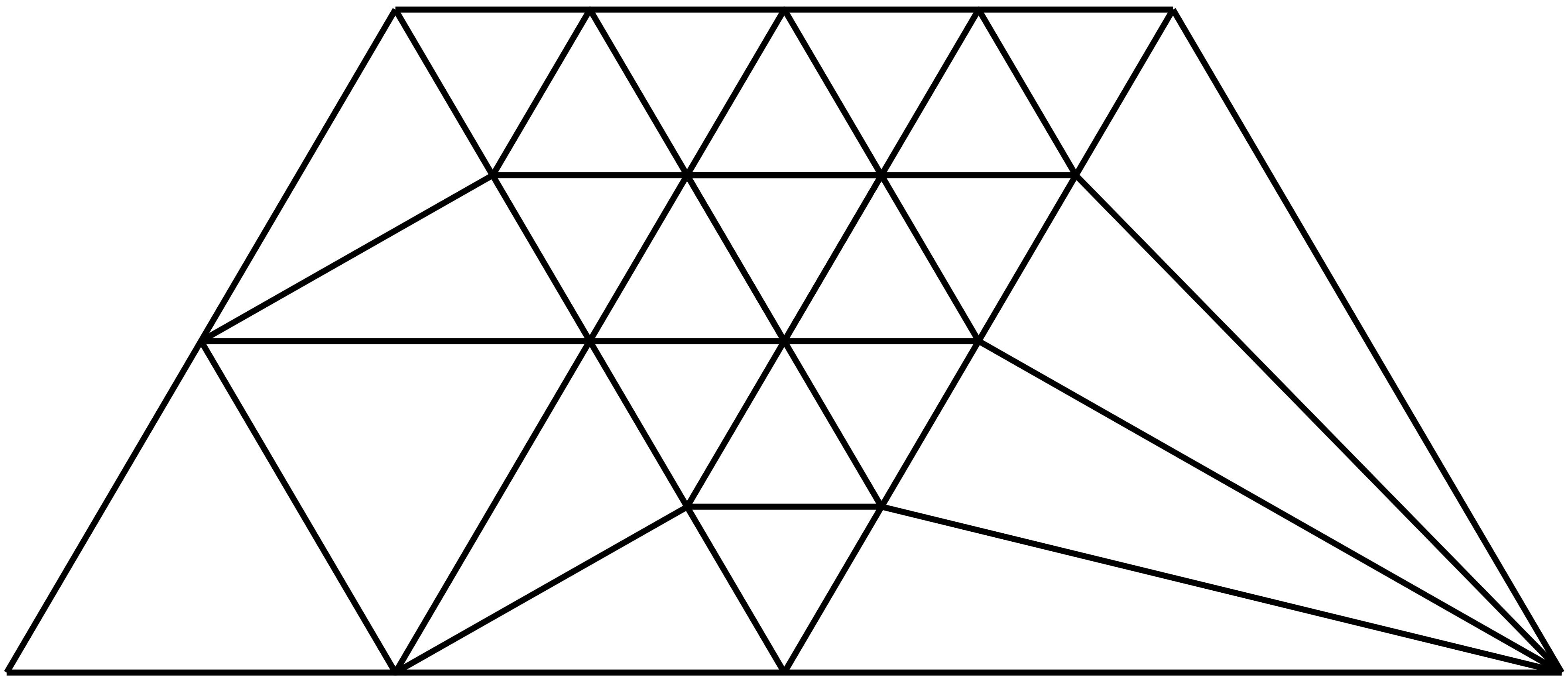
according to the needs
of the FEA





Element quality

The transition to repeatedly refined triangles should be smooth.



Smooth, element quality remains good, independant from the number of refinements.

Non-smooth, element quality may degenerate with the number of refinements.

The Marching triangle step

up to here: Each node is assigned to one and only one part.

To run a Finite Element calculation it is necessary that each element is assigned to one and only one part. (Every element needs a material discription.)

This switch is performed by a special refinement step.

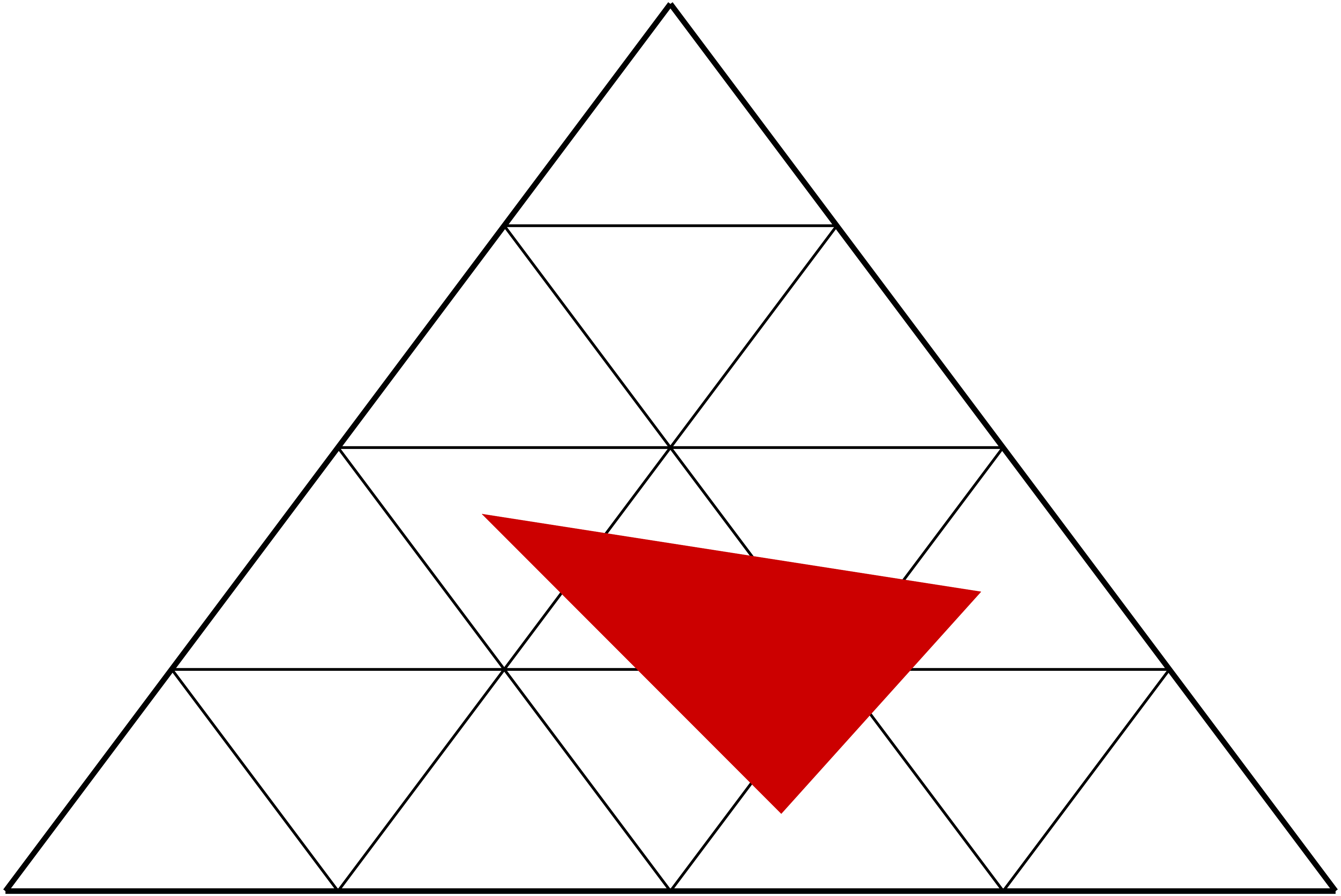
Goals

FE-boundary == part boundary , good element quality, few elements, refinement should terminate.

since part boundaries may be challenging (small details, recursive, not watertight) we reduce our requirements:

smart goal

refinement should terminate for simple parts.

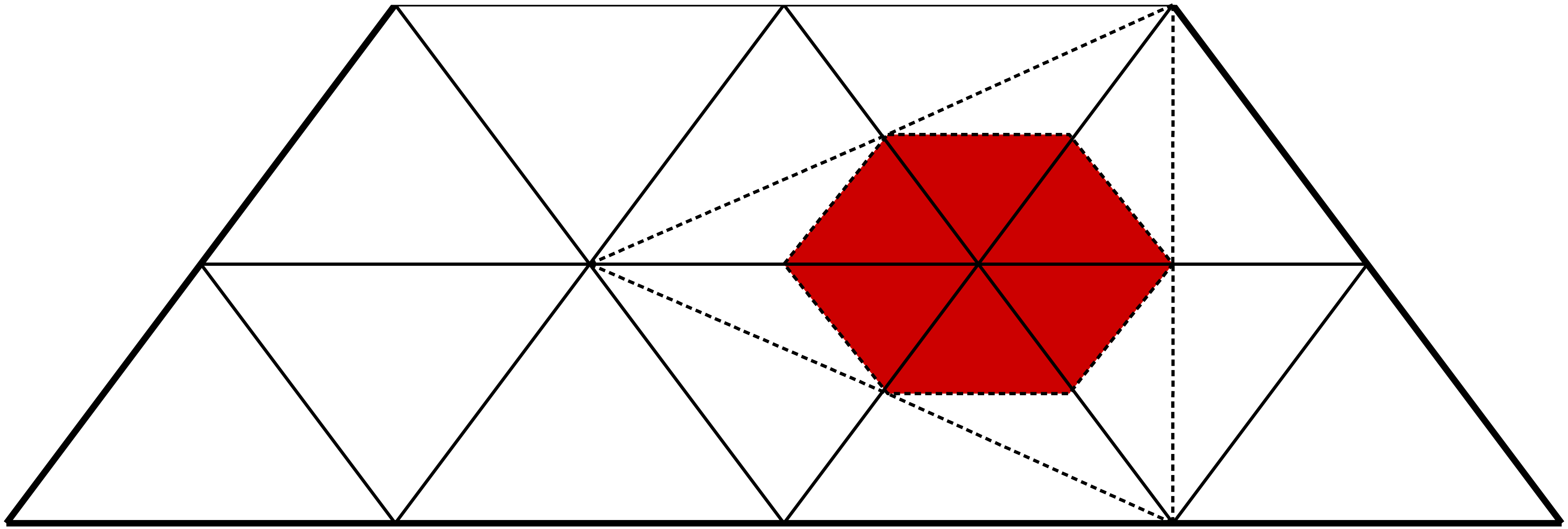


Marching Triangle :

mid-bisect edges with different part assignments (solid or fluid) at their nodes.
Fluid **and** solid is assigned to the new **nodes**.
Either fluid **or** solid is assigned to the new **triangles**.

Triangle quality is good, geometric quality is poor.
The triangle split is known from the BCC rfefinenent.

A

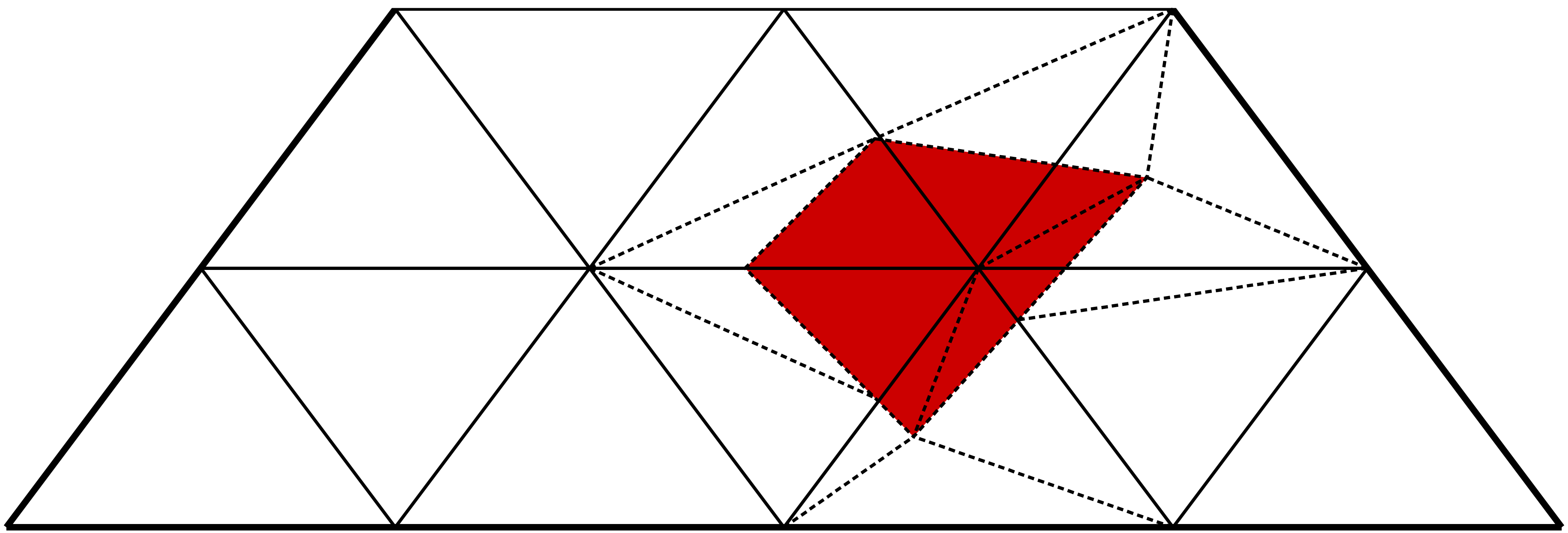


Marching Triangle:

B

Bisect edges with different part assignments at the edge-part crossing.
Create new nodes inside the triangle using the normals at the crossings.
Fluid and solid assignments as in A.

Triangle quality is worse, geometric quality is better than in A.
We need a new 1 to 5 triangle split.

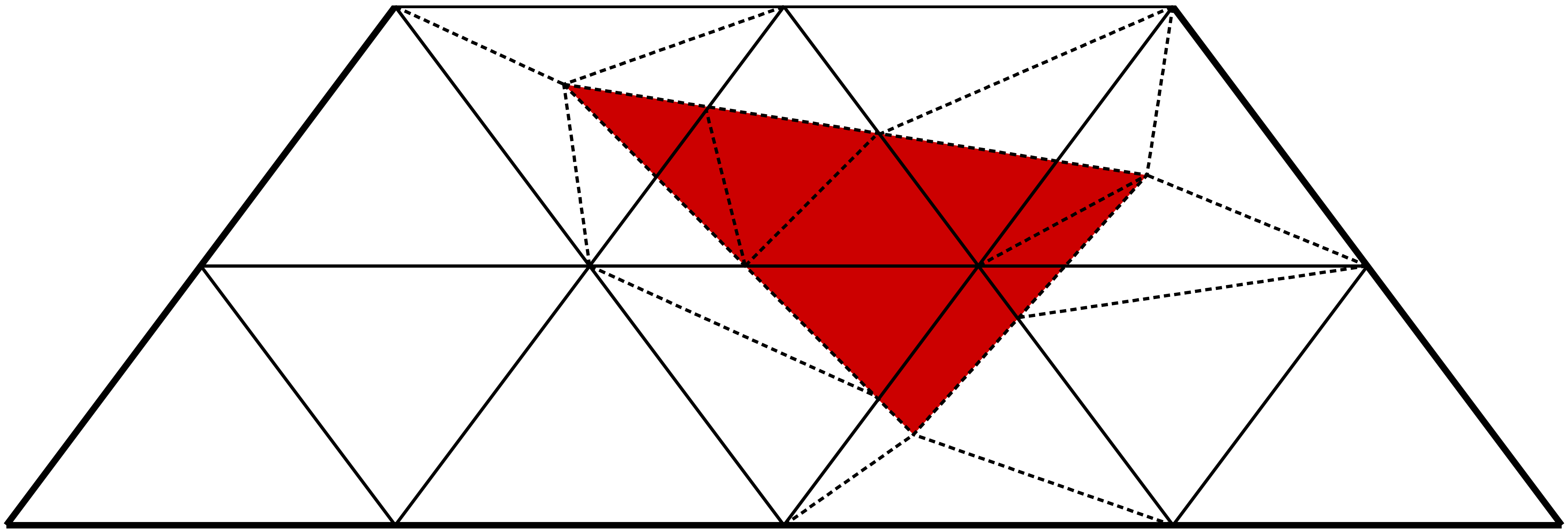


Marching Triangle:

Bisect edges which are single crossed.
Trisect edges which are crossed twice.

C

Triangle quality is as good as in B,
geometric quality is perfect.

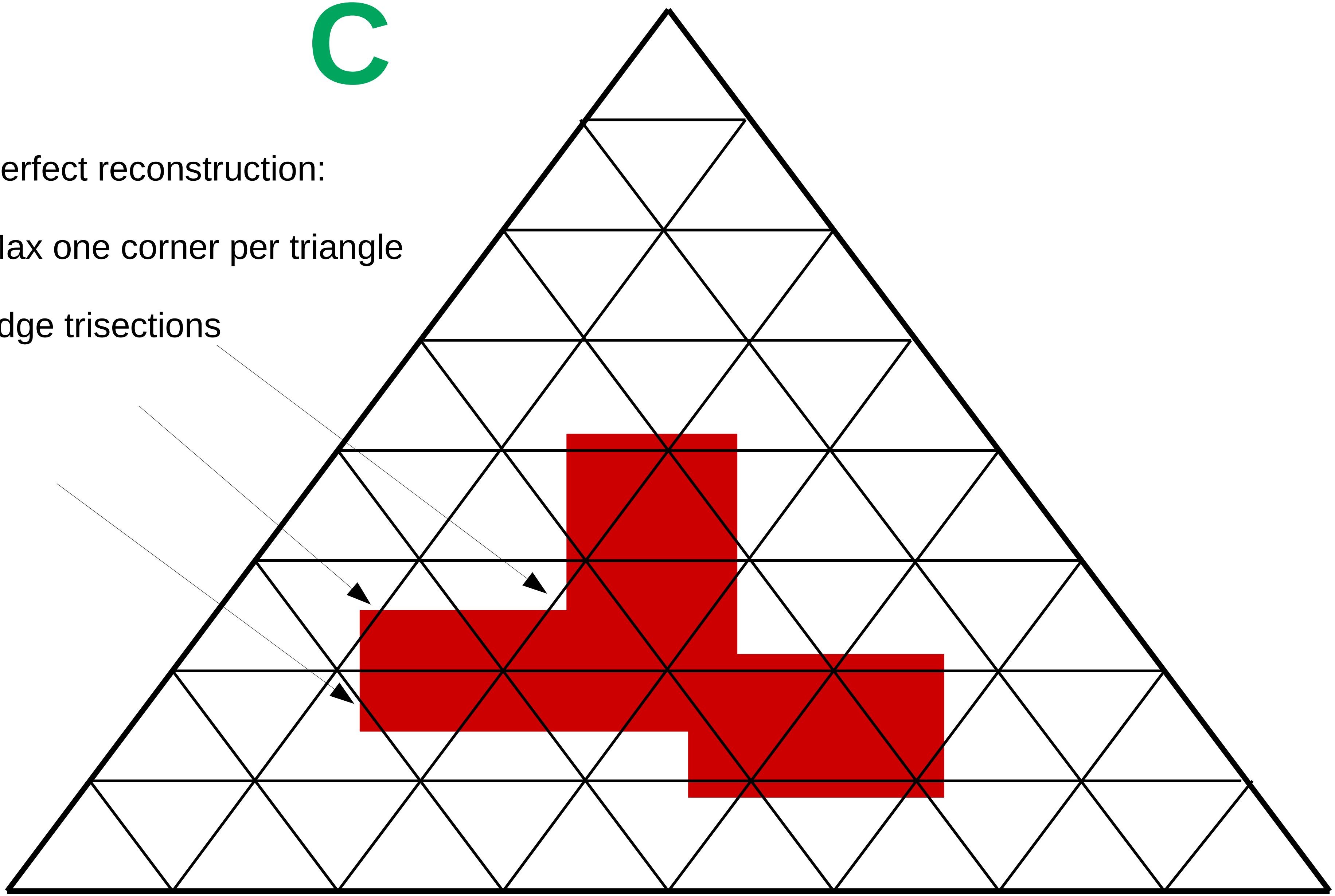


C

Perfect reconstruction:

Max one corner per triangle

edge trisections



The Marching Triangle method can be applied to any mesh.

The BCC refinement should give a mesh which is well prepared for the MT step.

Different MT methods need different refinement rules.

Element quality and approximation power of the MMT mesh:

Good results with bad elements

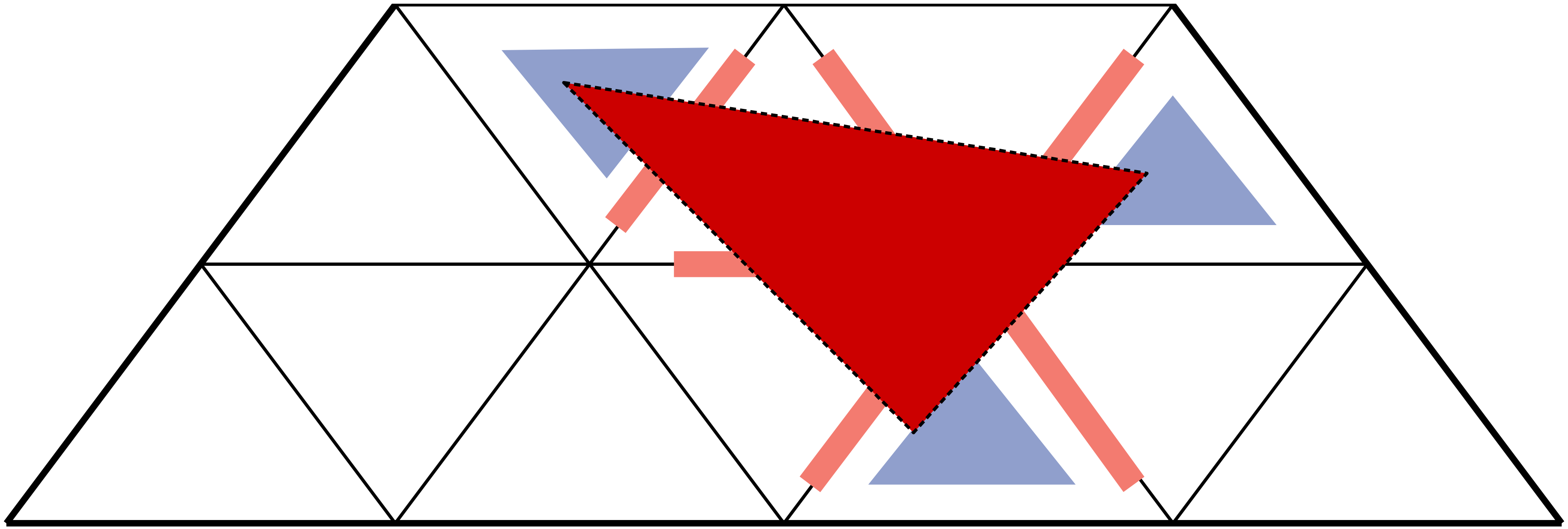
Element quality before the MT-step is good.
The MT step may be regarded as an enlargement of this Finite Element space.

Any enlargement does not spoil the approximation power of a vector space.

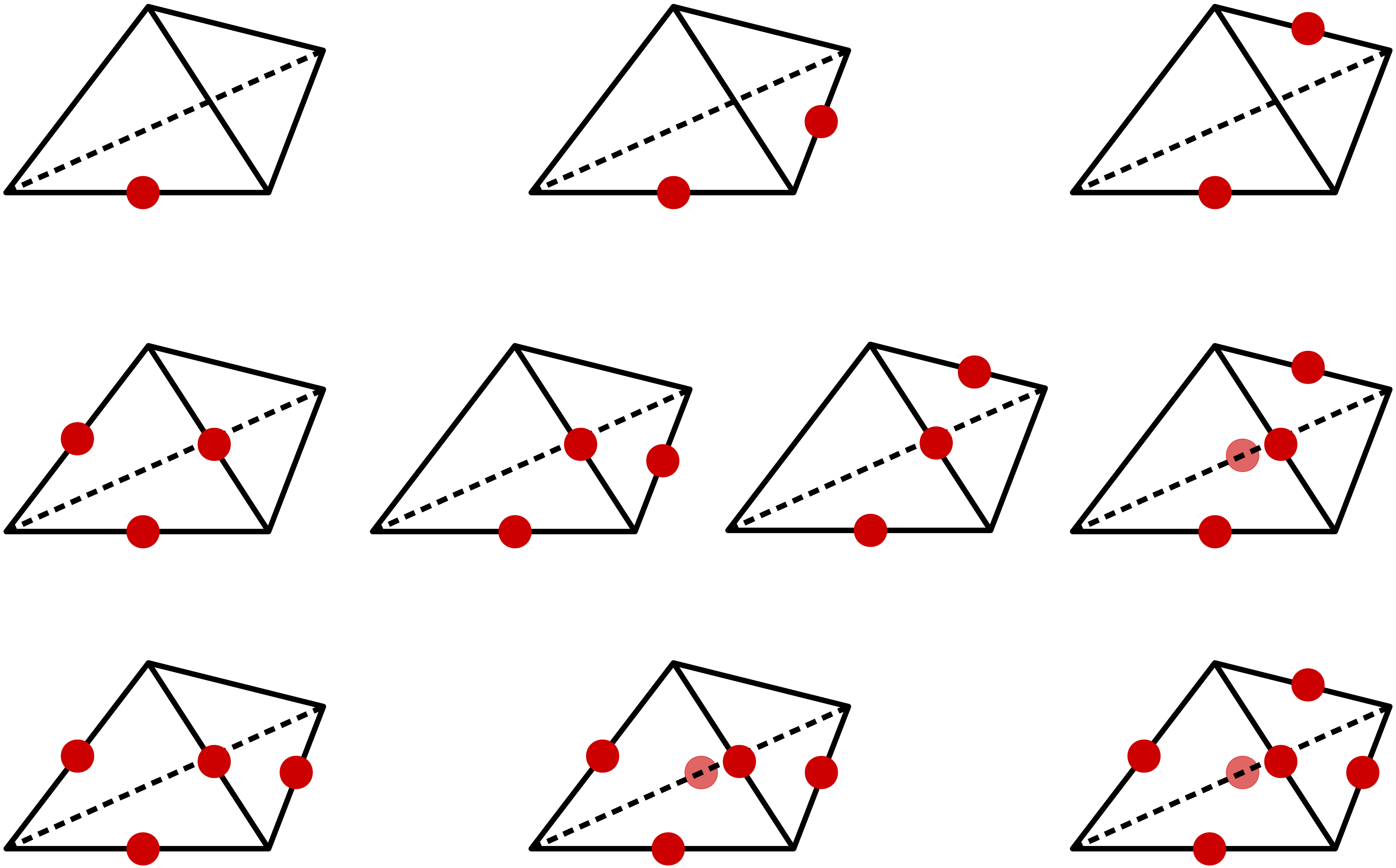
Element quality after the MT-step is bounded away from 0.0, independently from the parts to be meshed and from the number of refinements.
The lower bound of the element quality relies only on the limitation of the position of the new MT-nodes (= FE part surface) .

This limitation results in some small geometrical inaccuracies.

If new nodes lie inside some barycentric limits, the triangle quality can be bounded > 0.0 and the geometric quality is only slightly manipulated.

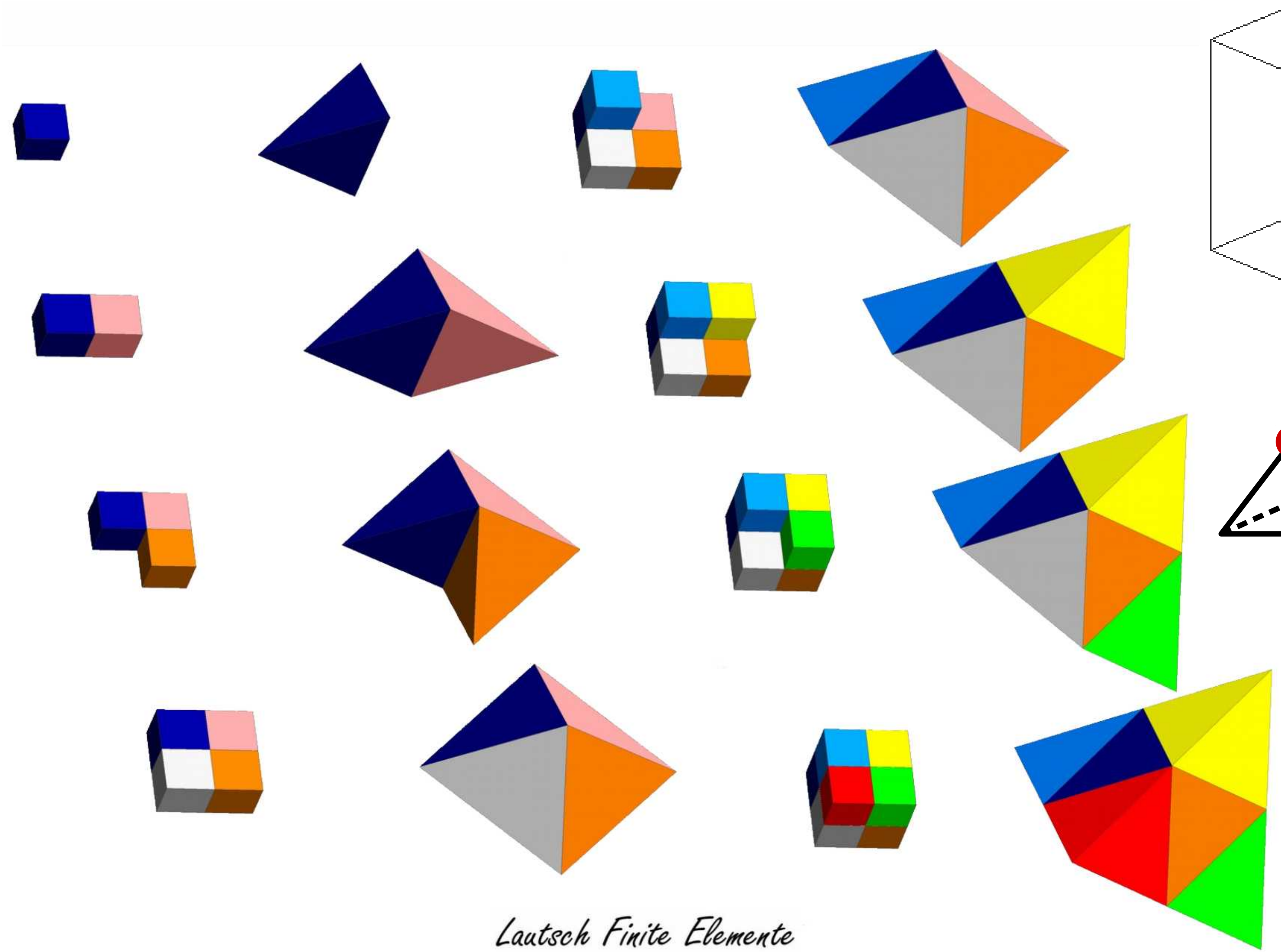


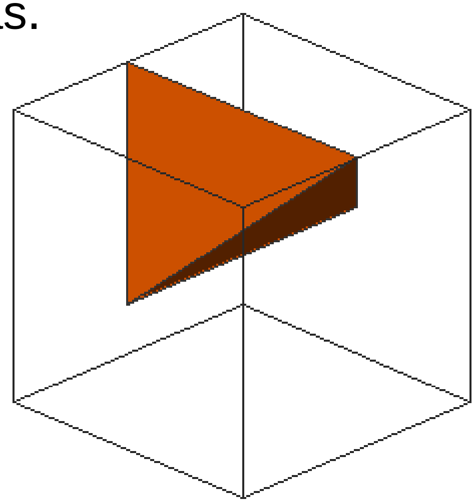
Transfer	**2D**	**to**	**3D**
Input: Part boundary	polygon		stl surface
Element quality	normalized volume / longest edge **2		resp. **3
Initial mesh	equilateral triangle		a single BCC tetra
Mesh refinement	quadtree		octree
	3 Triangle splits		10 Tetra splits
Geometry features	corner		edge
	–		corner (n edges meet, convex, concave)
Refinement terminates for simple parts	Marching Tetra **C**		under construction
Mesh improvement		edge collapse, edge swap	

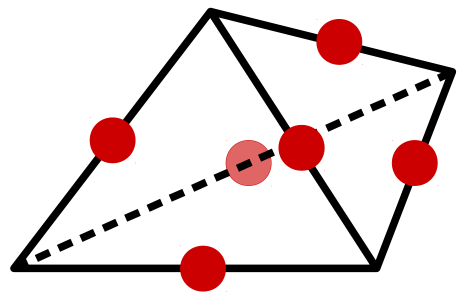


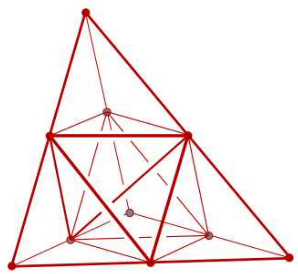
10 tetra refinement patterns: 1 1-cut, 2 2-cuts, 3 3-cuts, 2 4-cuts, 1 5-cut, 1 6-cut

The BCC Tetra can be split to 8 tetras of the same shape: Remove 4 tetras at the corners and split the remaining octahedon at the shortest diagonal into 4 tetras.









Gallery

Lautsch-fe.com/gallery

	number of parts	FEA	
Recursive car	inf	--	
Bike	634	linear static, ANSYS	
Disc brake assembly	18	heat, thermal expansion, eigenforms, MARC	
Knife	1	eigenforms, PERMAS	
Windmill	1	--	IMR 2023 meshing contest
Blue crab	1	--	IMR 2024 meshing contest

Outlook

Release 7	2021	multi part	rough wrapping
Release 8.5	2023	one part	2 stage bisection wrapping
TODO: Release 9		one part	1 stage trisection wrapping
TODO: Release 10		multi part	
TODO:	FEA - adaptive		
TODO:	nonlinear problems		
TODO:	fast linear solver		
TODO:	parallel		
TODO:	transient problems		
TODO:	4D		

A BCC tetra is cut by the surface of a gear wheel