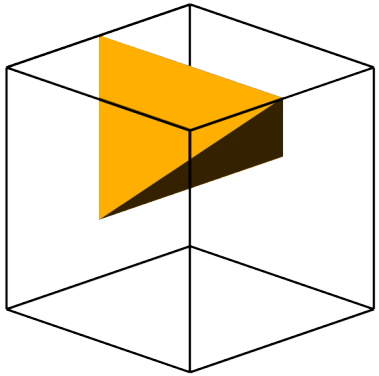


**build a convergent hierarchy of**
**Finite Element meshes.**

18.2.2026

Sorry, but FE needs some Mathematics (greek, = art of learning). Please help me to make it understandable for everybody, who is interested.

Temperature or displacement is computed via

Boris Galerkin

$$: A\ x = b$$

$\int$ Material properties and functions

Integrals are calculated over Finite Elements, a unique material description is required.

Areas with a common material description are called parts.

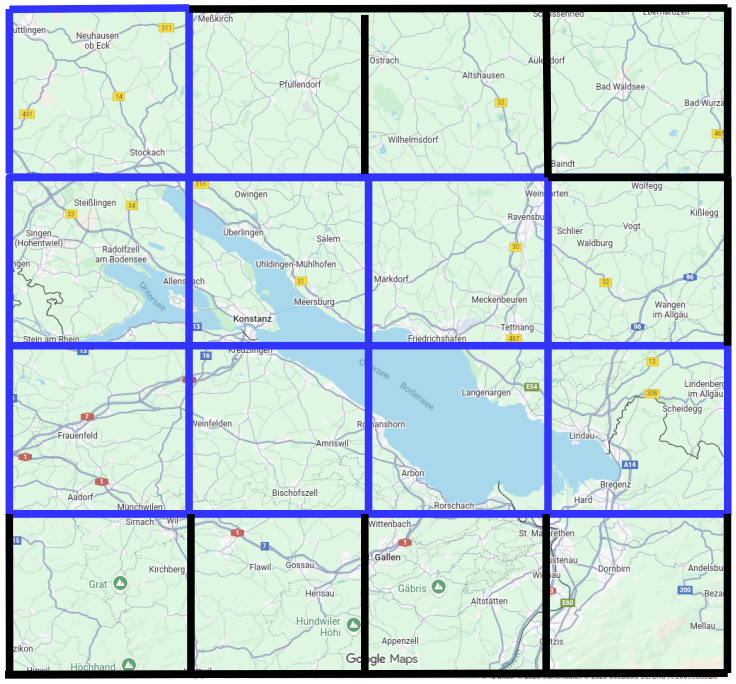
We start with a single element which covers all parts and we refine this element adaptively. The critrion is:

REFINE MULTI PART ELEMENTS
TO SINGLE PART ELEMENTS

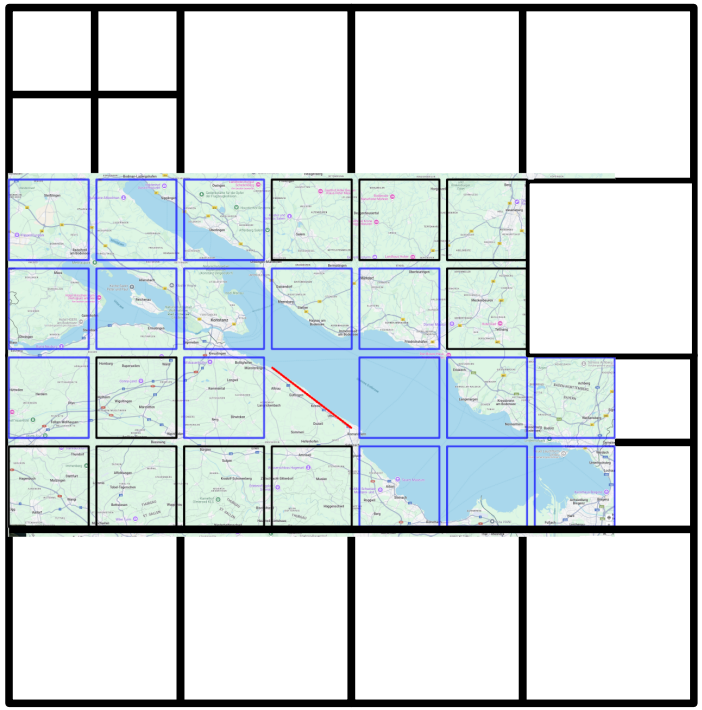
Initial

1st

2nd



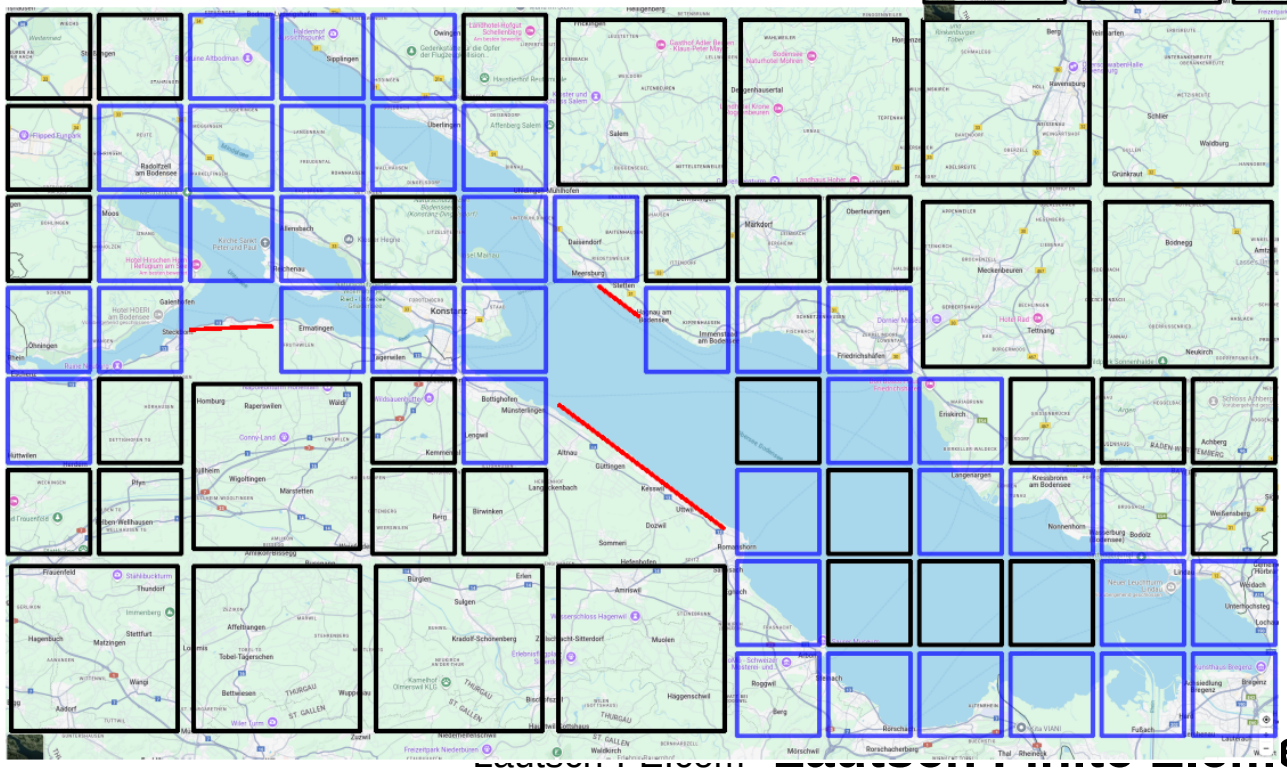
3rd refinement



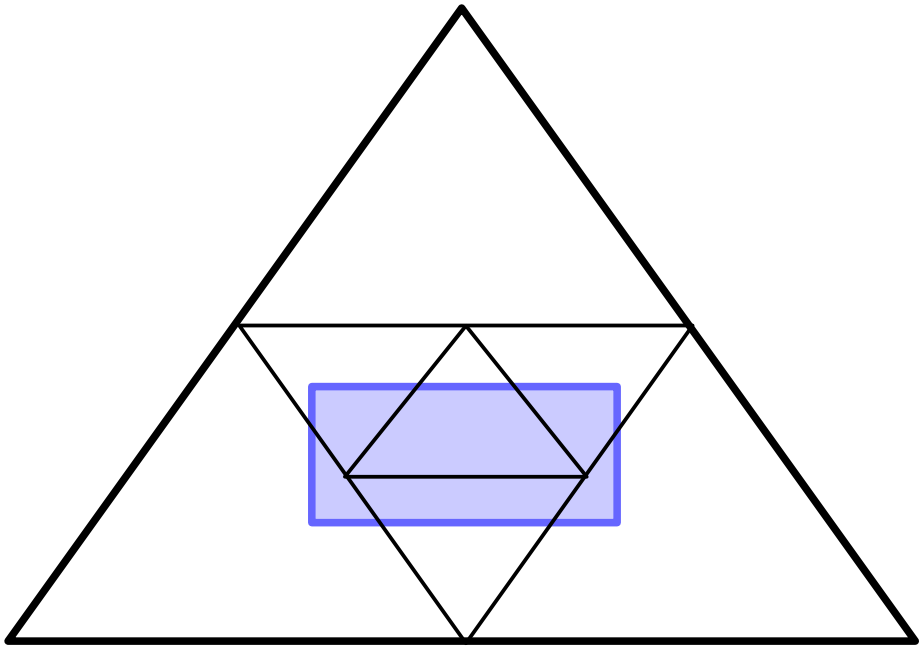
Black quads are single part elements
- fluid or solid

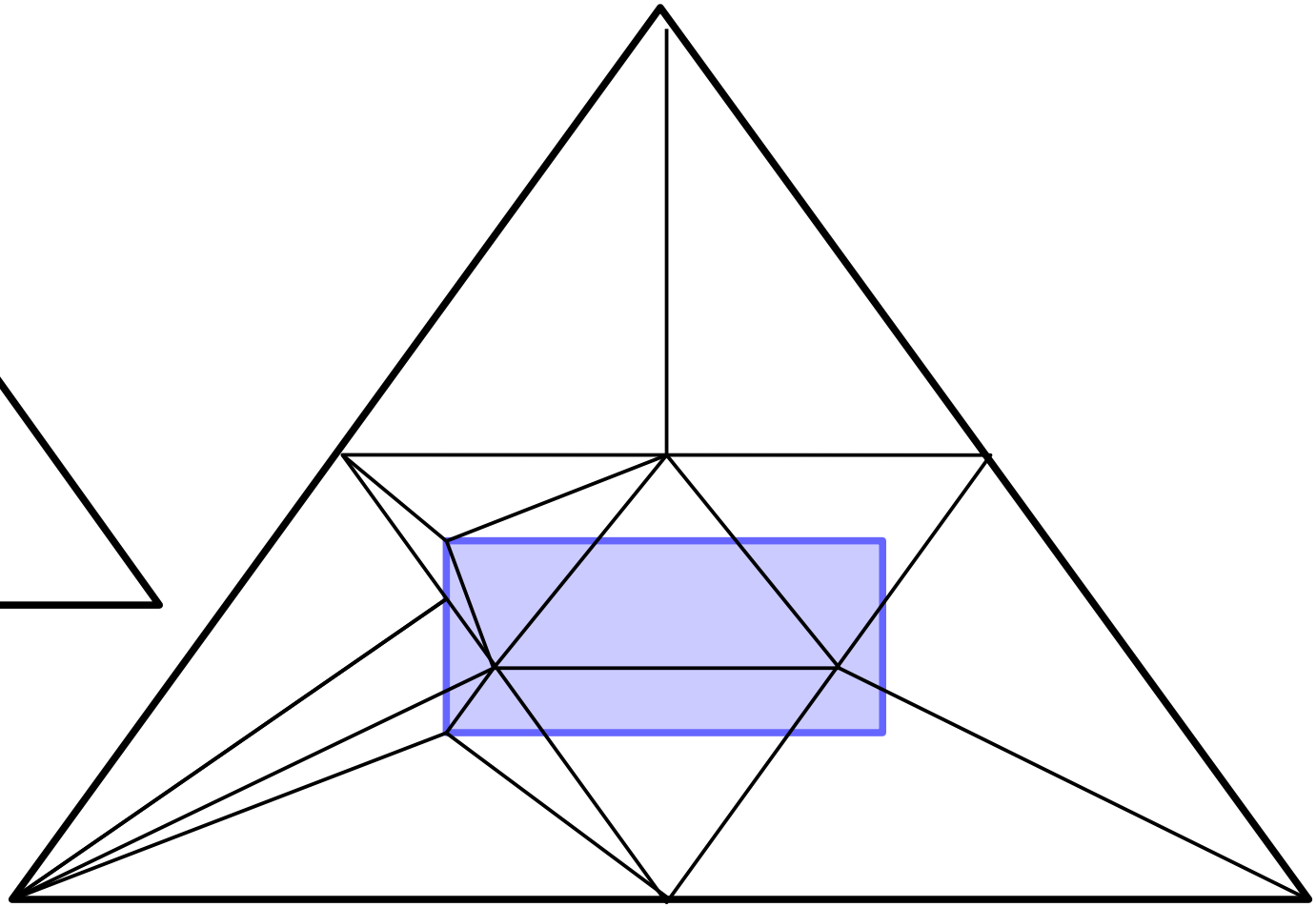
Blue quads are multi part
- fluid and solid

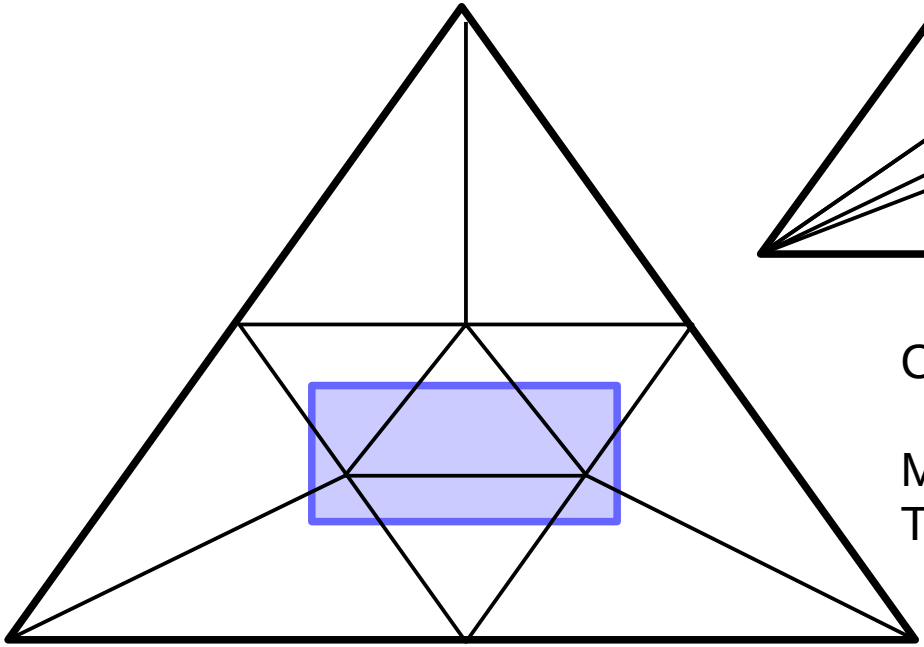
A blank quad with a red line means:
Ready to run a Finite Element job



4 th refinement of
Blue (multi part) elements







Conformal triangular mesh, edge bi-and trisection.

Mesh refinement terminates for simple parts.
The corners of the blue part are reconstructed correctly.

# Multigrid Marching Tetra Method

Refine the BCC (background) mesh to build a convergent hierarchy of Finite Element meshes.

A: Decide for BCC mesh elements: single part or multi part

B: Build a Finite Element mesh : **REFINE B** the BCC multi part elements and obtain one part elements.

C: **REFINE C** the BCC multi part elements, when **REFINE B** has failed

goto A

The algorithm terminates naturally when there are no problems in **REFINE B**.

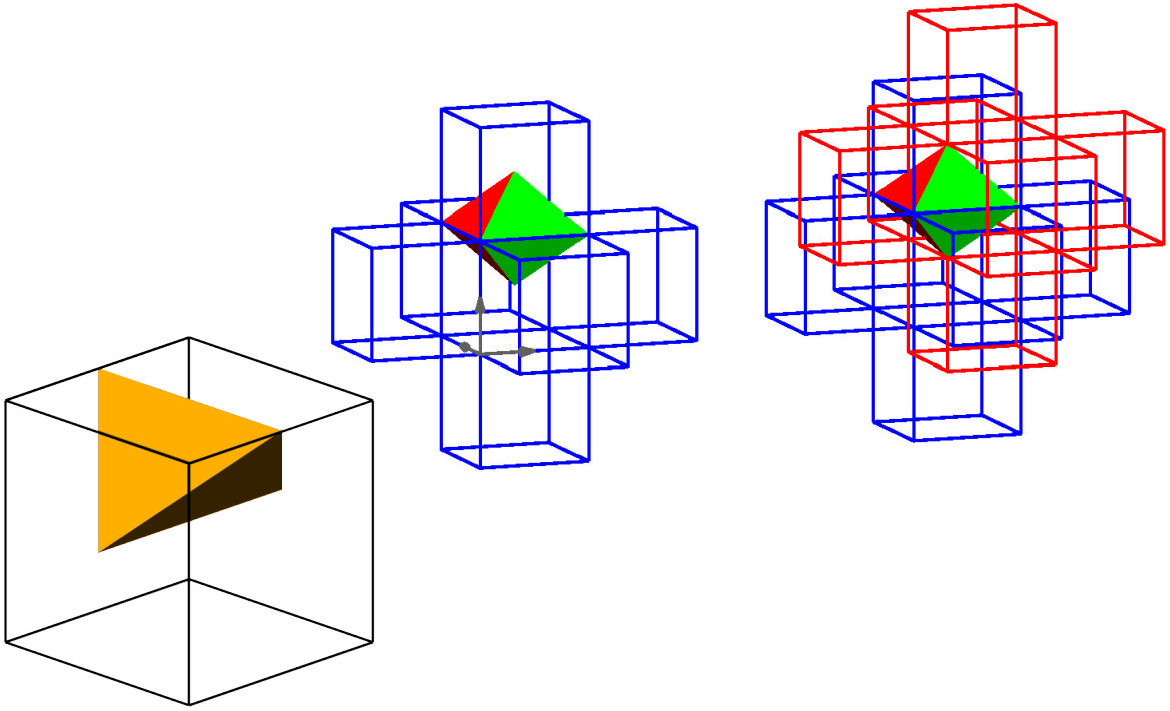
## Do not refine the Finite Element mesh.

In 3D space we rely on a special Crystal grid: Body Centered Cubic grid – BCC.

There are two ways to describe this grid:

A: Take a simple cubic mesh  and add a  ( 0.5, 0.5, 0.5 ) translated copy.

B: Take a cubic tetra and split this tetra to 8 selfsimilar cubic tetras.



4 methods of **REFINE B** to create single part elements from multi part elements.

0 just skip and ignore.

1 mid edge bisection

2 edge bisection

3 edge bi and trisection

1-2-3: Edge splits define triangle splits. Triangle splits define tetra splits.

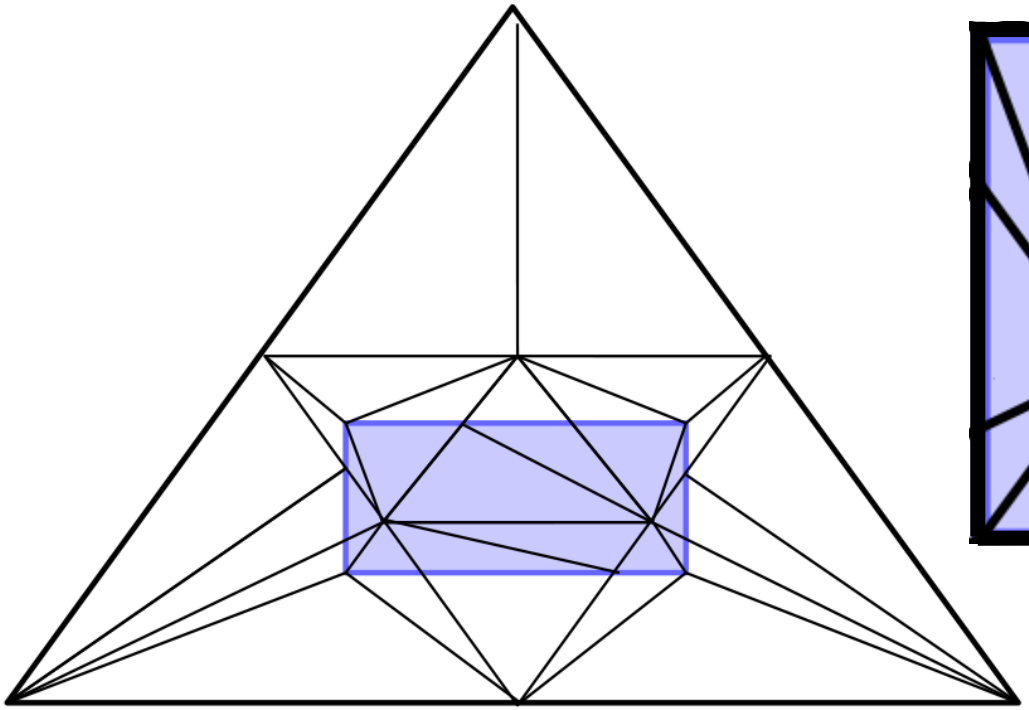
| | REFINE A | REFINE B3 |
|---|---|---|

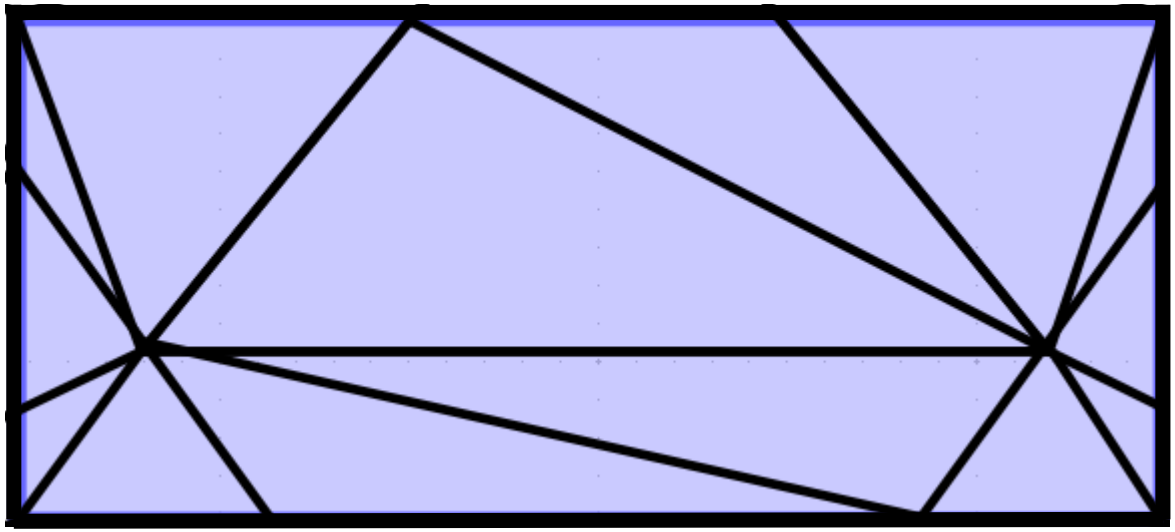
Nodes ( not the initial 4 nodes) are given by their barycentric position in terms of prior edges / triangles / tetras

| | midnodes on edges | free on edges / triangles / tetras |
|---|---|---|
| Edges | bisection | bisection and trisection |
| Triangles | 3 cases | nn cases |
| Tetras | 10 cases | nnn cases |
| * worst tetra | 0.16 | 0.0 |

* Tetra quality = constant * Volume / max edge **3 . Best = 1.0

**REFINE B3:** To avoid 0.0 quality elements
- run mesh improvement or
- consider bad elements as vector space enrichment.



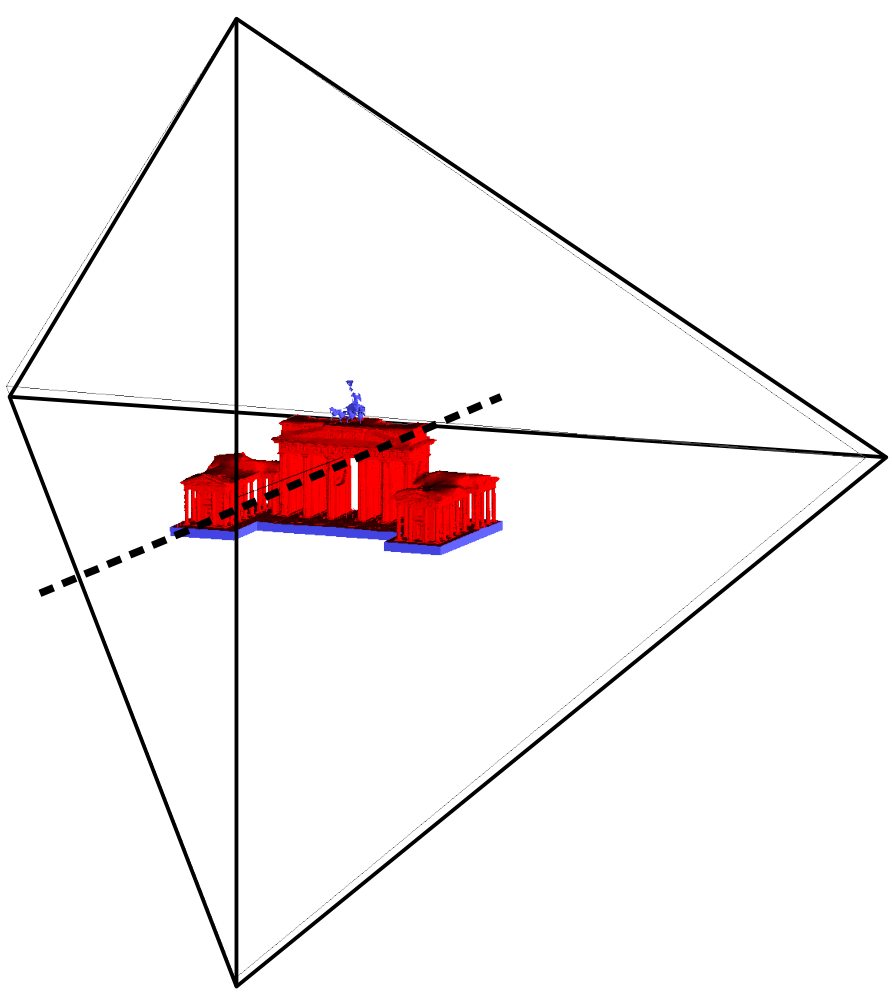


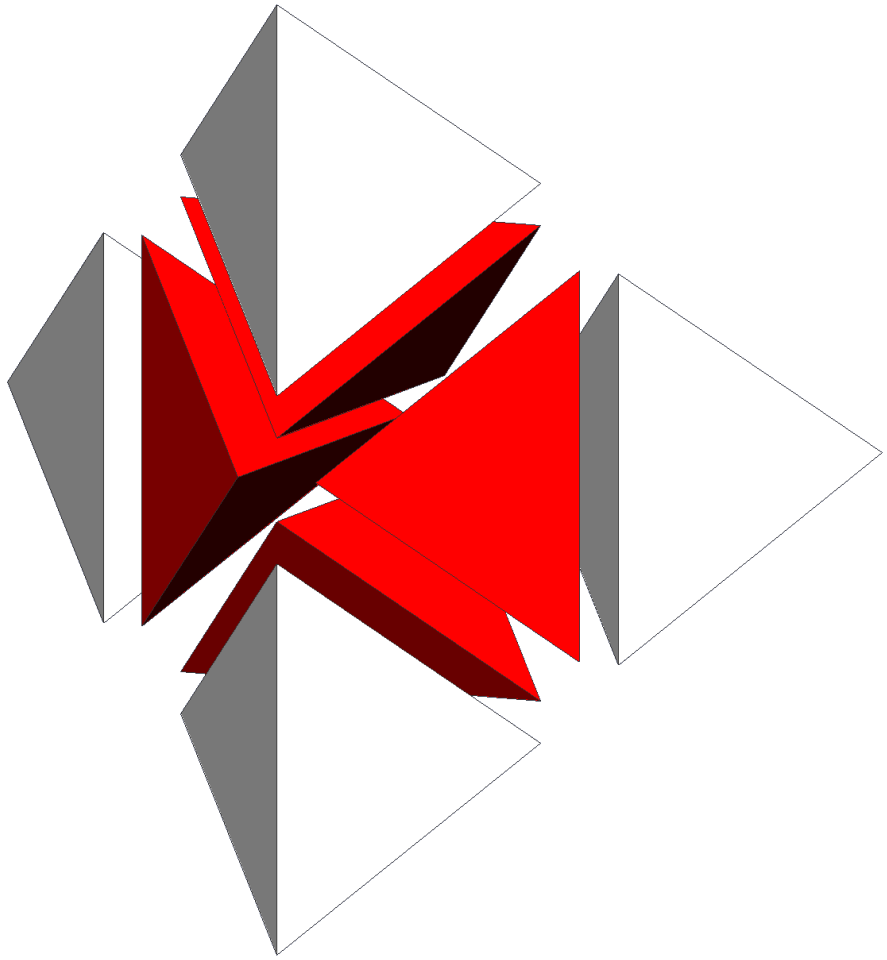
Conformal triangular mesh, edge bi-and trisection.

Mesh refinement has terminated for a simple part.
The corners of the blue part are reconstructed correctly.

Edge collapse may improve the mesh quality of the blue part.

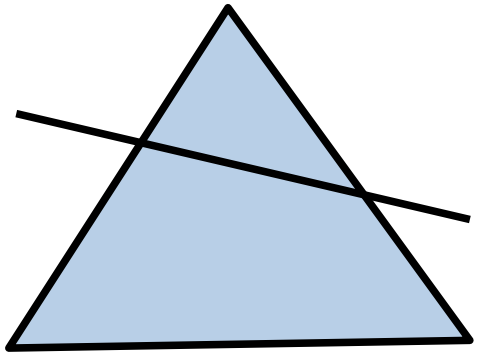
Usually the first edge cut fails: more than 2 cuts on the first new edge, the adjacent triangles cannot be split according to the rules ( 201-1) , hence the red tetras have to be split  1 to 8



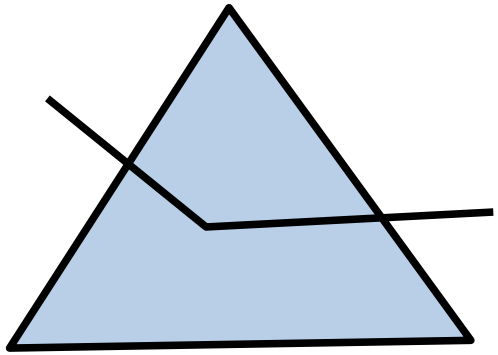


Edge bisection refinement patterns for triangles and tetras

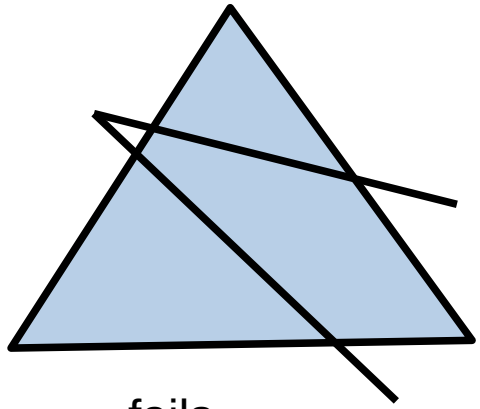
abc-d
a edges 0 cuts
b edges 1 cut
c edges 2 cuts
d corners



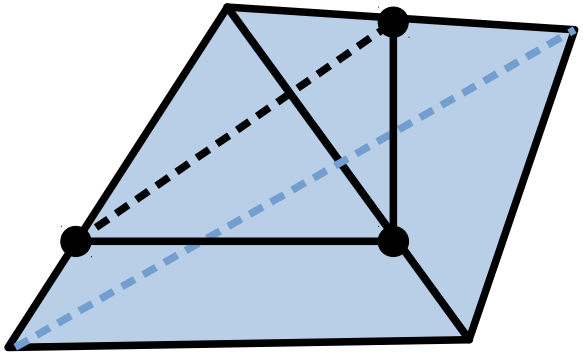
120-0



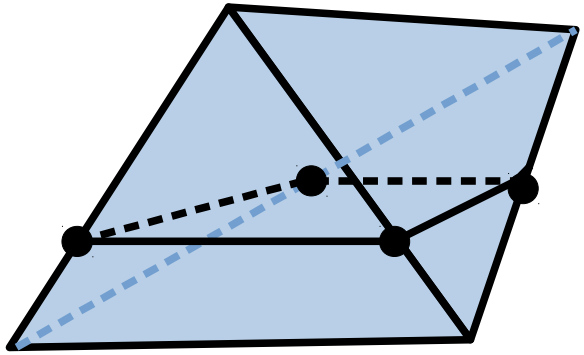
120-1



fails



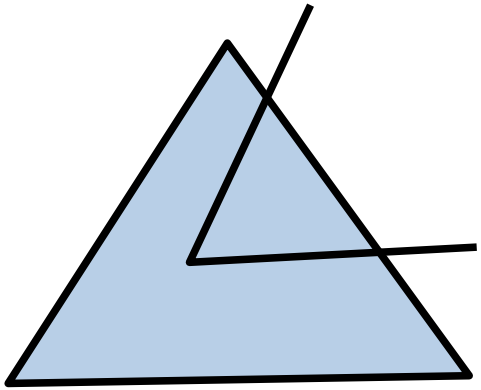
330-0



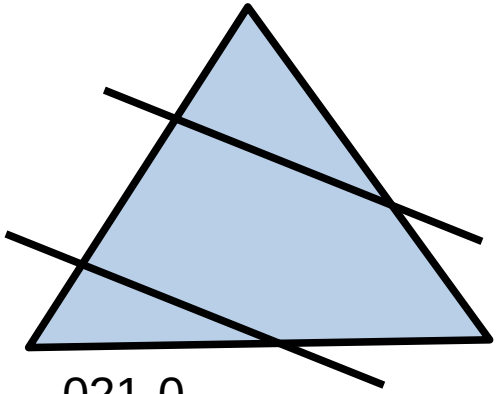
240-0

240-1, 240-2, ...

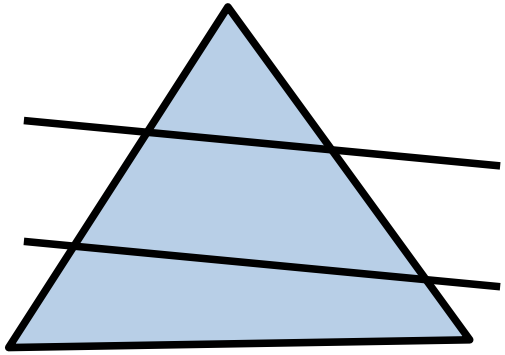
Edge trisection refinement patterns for triangles



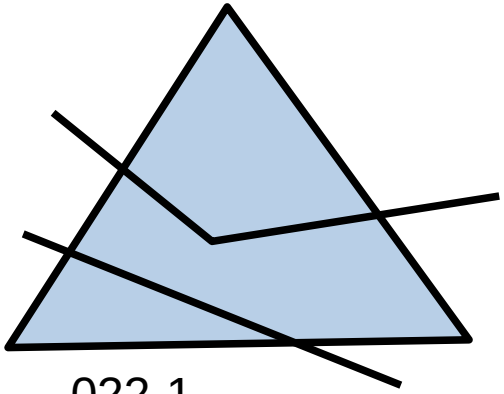
201-1



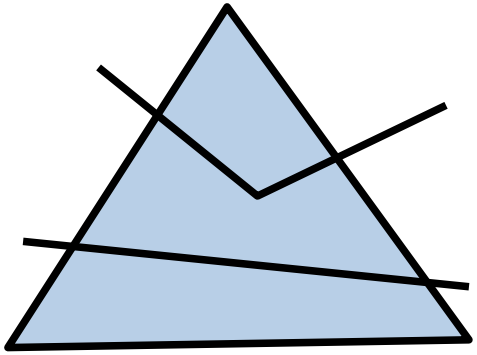
021-0



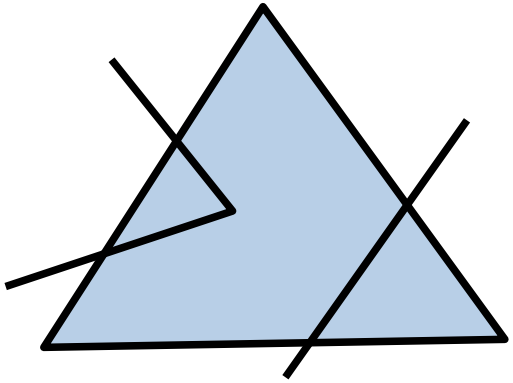
102-0



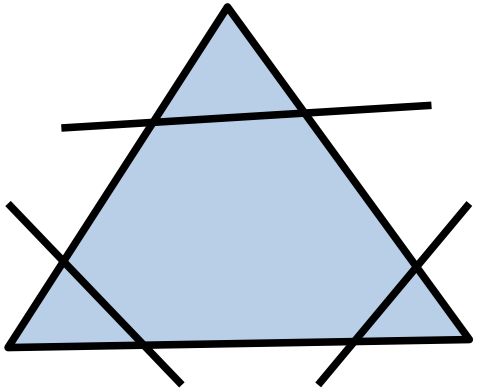
022-1



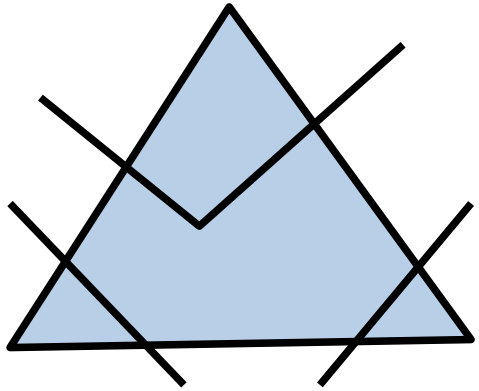
102-1



021-1

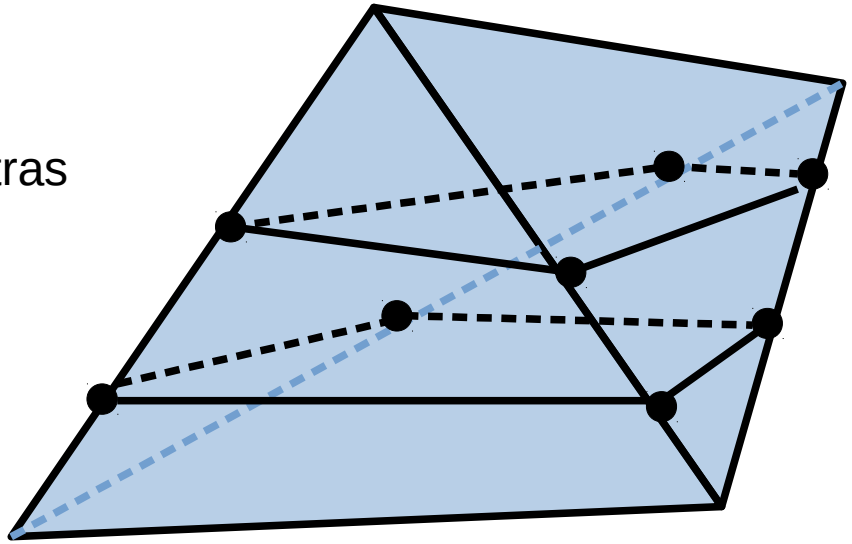


003-0



003-1

Edge trisection refinement patterns for tetras

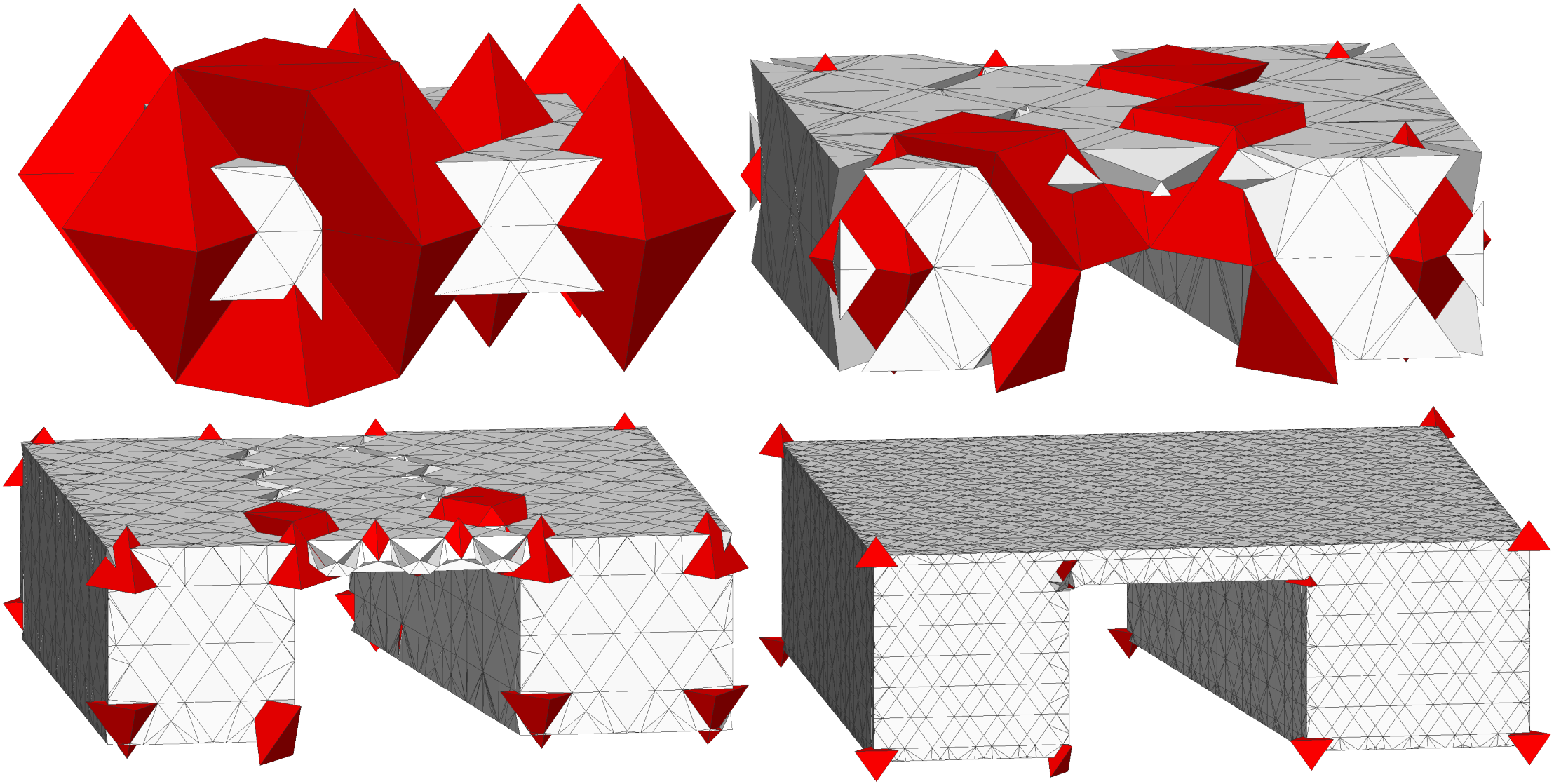


204-0-0

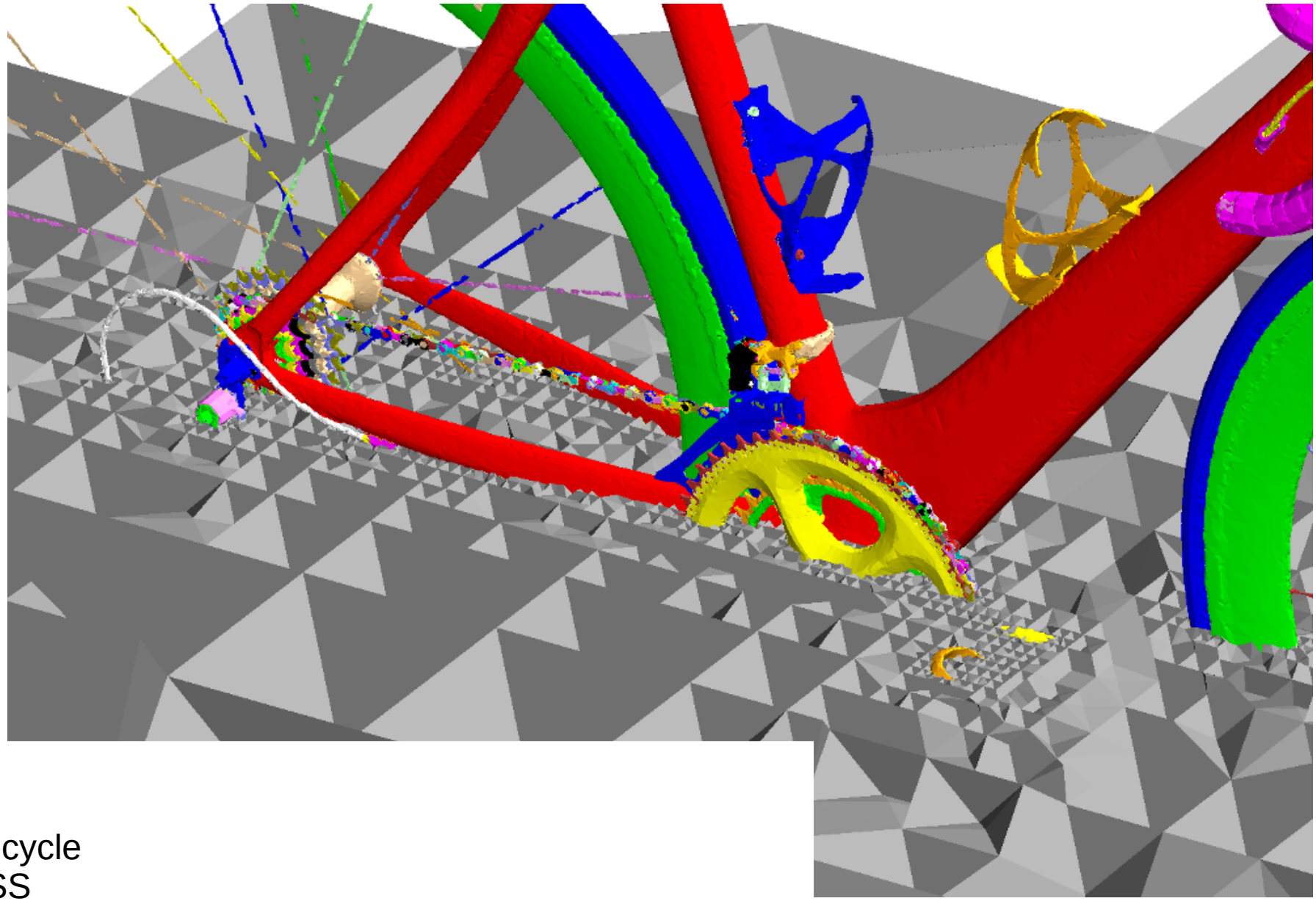
The last digit for 3d corners.

More than 50 patterns.

Very bad tetra element quality may occurr here. Later we discuss that this is not a serious drawback.



Red tetras are multipart, global refinement, level 3 to 6



Multi part

634 parts bicycle
100 parts ISS
3 parts IMR 26 Brandenburg Gate: Gate, Quadriga and fluid.

Adapt the bi+trisection rules ...