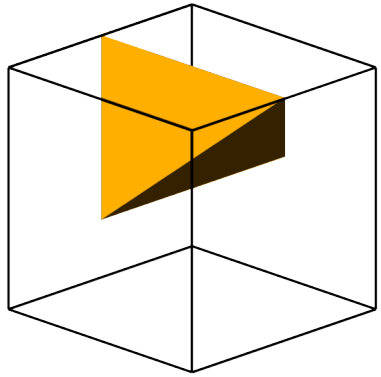
Lautsch Finite Elemente GmbH



Bisection and trisection of edges cut by part-geometry

THE MULTIGRID MARCHING TETRA METHOD

A single BCC Tetra covers all parts

3 times octree refinement

Loop: local refinement:

- octree, midnode, edge bisection for tetras which are selected by the refinement criterion

Keep element quality > 0.155 by adding edge bisections

Final local refinement step:

Each node is assigned to exactly one part

Each tetra is assigned to exactly one part

Keep element quality > 1.e-7 by barycentric limits or other means

MT refinement of tetras with different part properties at its nodes.

We discuss different kinds of the final step i.e. the MT method
Each MT method owns its specific criterion for the local octree refinement:

A: The simplest method (best element quality) is just to
remove tetras with different part-assignements at their nodes.

Refine Tetras with different part-assignements at their nodes.

B: A second simple method is to consider edge bisections only.
One triangle pattern, two tetra patterns. These patterns cannot fail.

Refine Tetras with different part-assignements at their nodes.

C: A third method is to consider the surface geometry
at the edge splits. Triangle and tetra splits may be kinked.
These patterns can fail.

Refine Tetras which will fail.

D: The next method starts with edge bi- and trisections…..

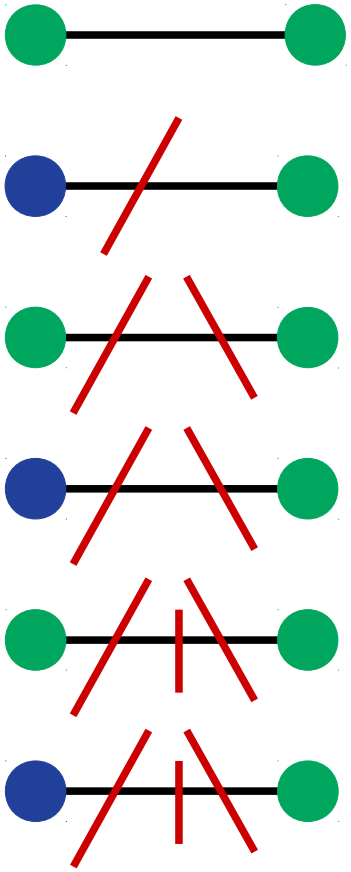
Refine Tetras which will fail.

… more than 1 part ...

1D edge 3 patterns: 0 1 2 geometry cuts

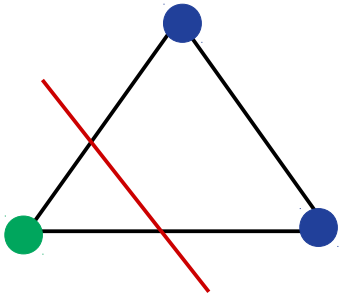
→ 2D triangle patterns

→ 3D tetra patterns

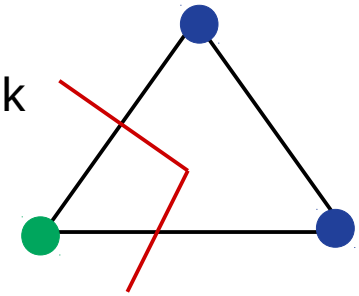


Bisection	Trisection
ok, because of same material	
ok	
fail	ok
fail	fail
fail...	

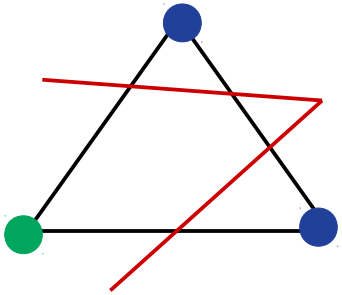
Bisection 110



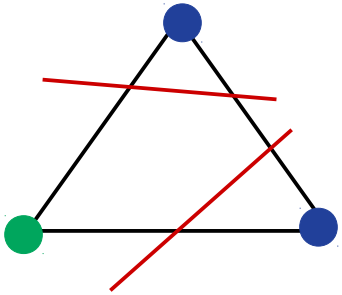
110 + kink



fails, if the kink is outside of the triangle.

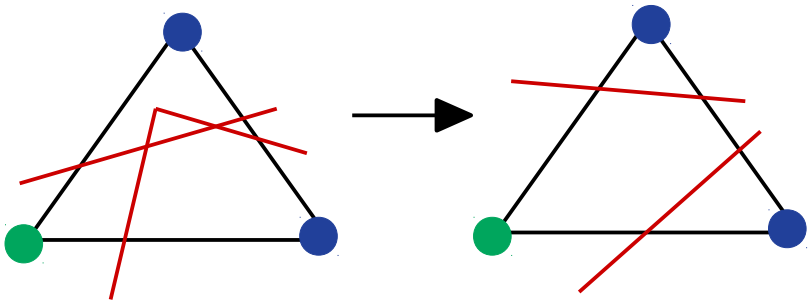


This failure is healed when we allow edge trisections.



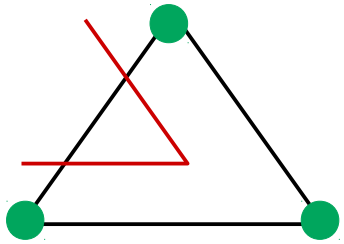
We then have to consider some more refinement patterns, which of course can fail in their own way.

The stopgap solution is to ignore the kink.

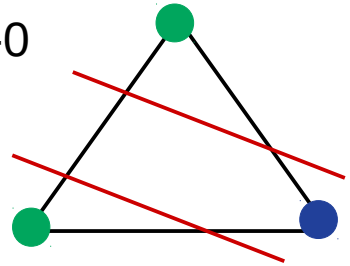


Trisection

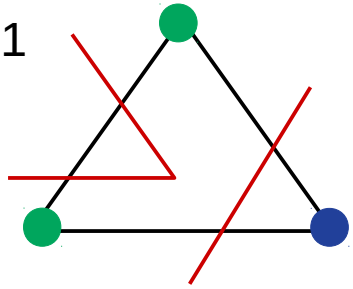
200

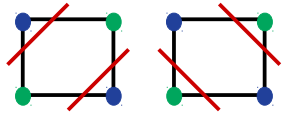


211, ambiguous 211-0



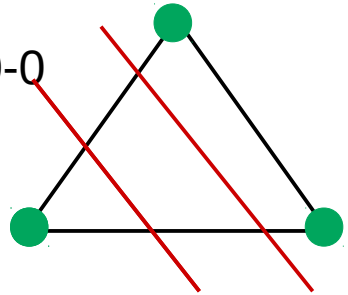
211-1



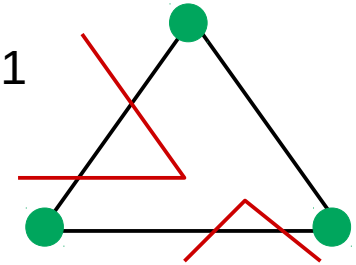


similar to the quad case

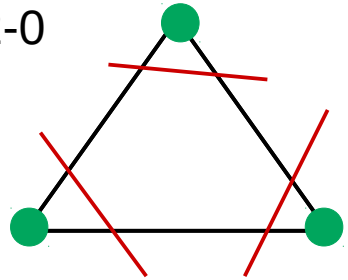
220, ambiguous 220-0



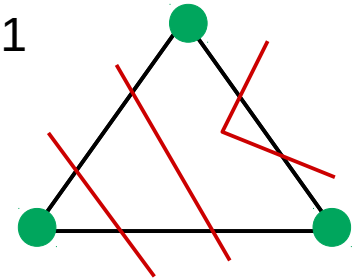
220-1



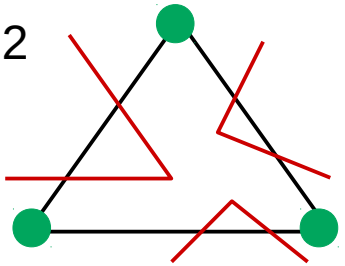
222 3 cases 222-0



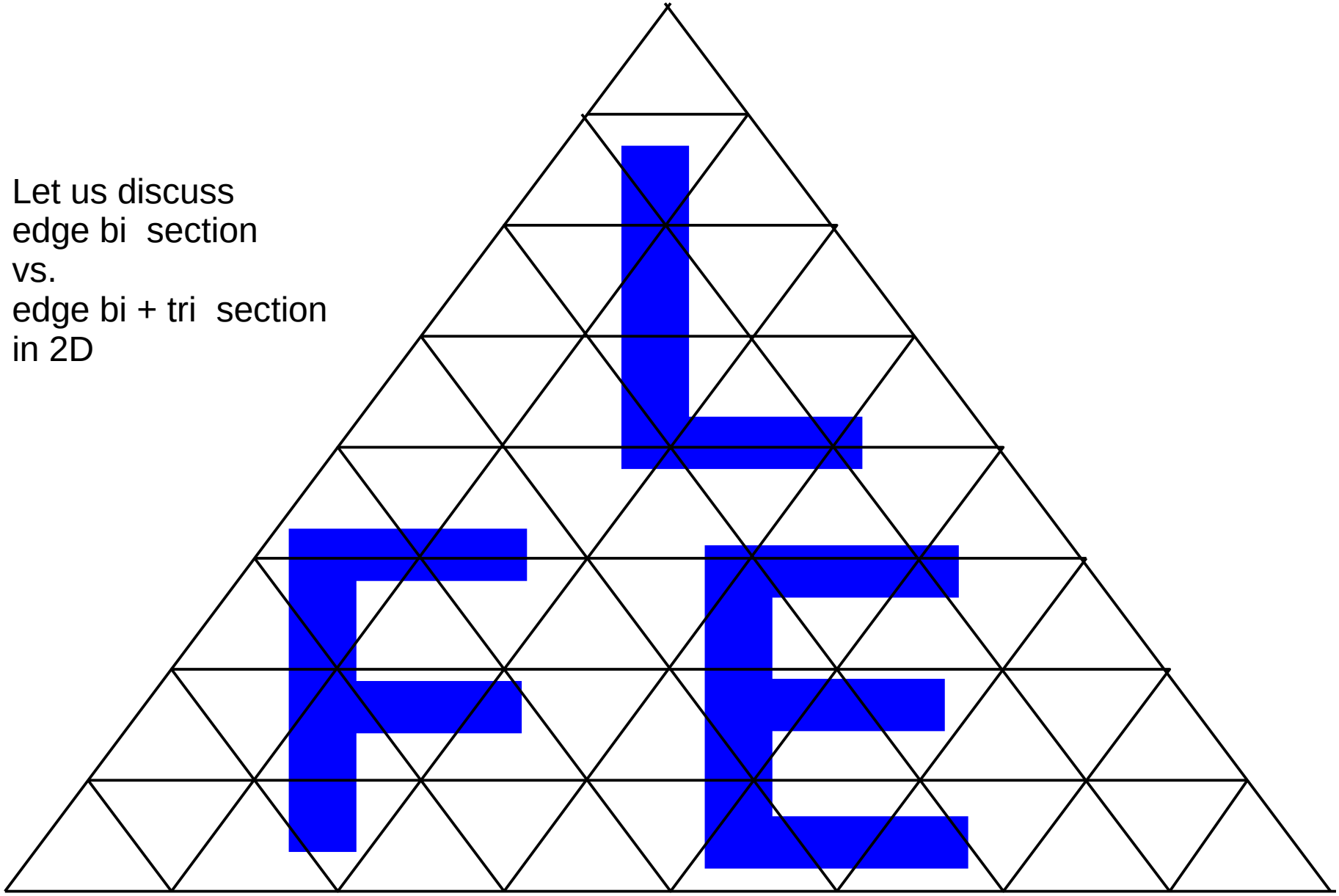
222-1



222-2



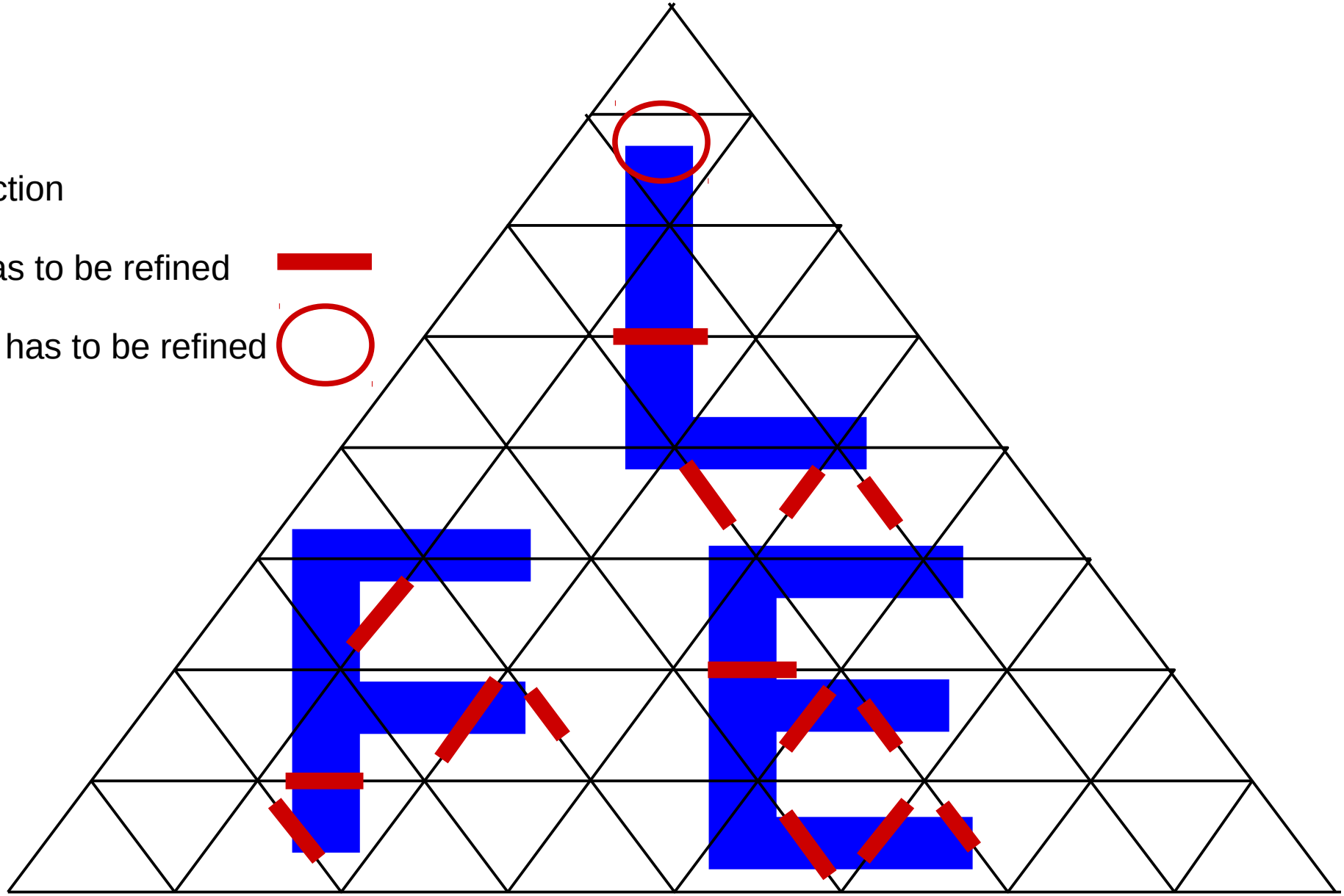
Let us discuss
edge bi section
vs.
edge bi + tri section
in 2D



Bi - section

edge has to be refined

triangle has to be refined

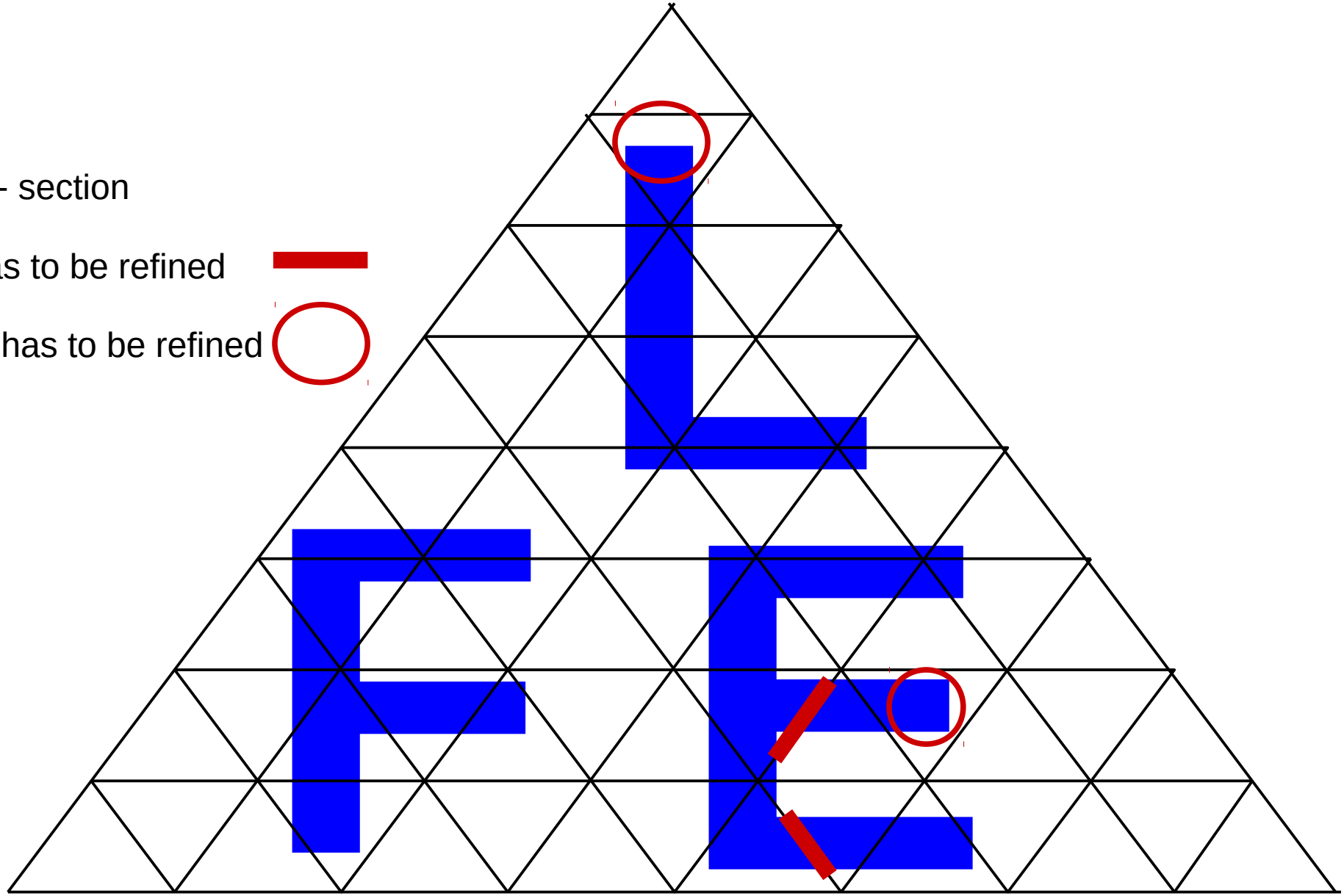


Bi - section

Bi + Tri - section

edge has to be refined

triangle has to be refined



Bi + Tri - section

A rough criterion for 2D bi- and tri- section:

A polygonal bounded part is reconstructed correctly when every triangle does not contain more than one corner.

2D MMT - Bi + Tri – section refinement terminates for simple parts.

The tetra refinement patterns are the logical extension of the triangle refinement patterns on the tetra surface.

3D

We omitt the kinked triangle patterns now to give a shorter overview of the tetra cases.

We observe these cases:

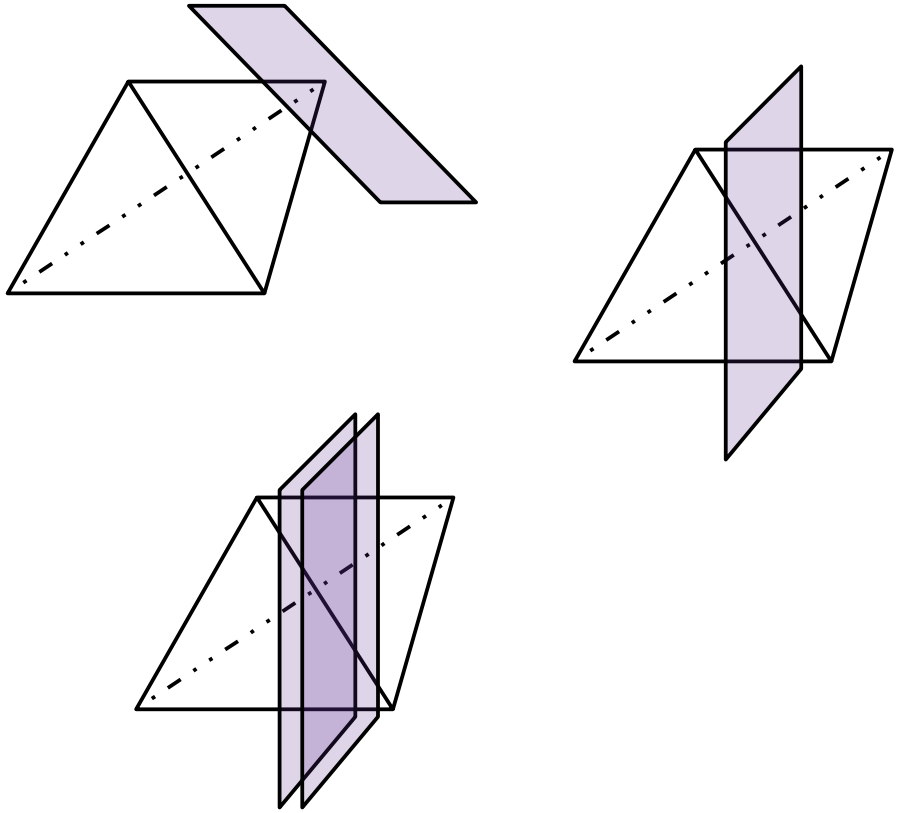
110 110 110 0
110 110 110 110

200 200 0 0
200 200 200 200
220 200 200 0

220 220 220 220
220 220 220 0

222 211 211 211
222 220 220 110

222 222 222 222



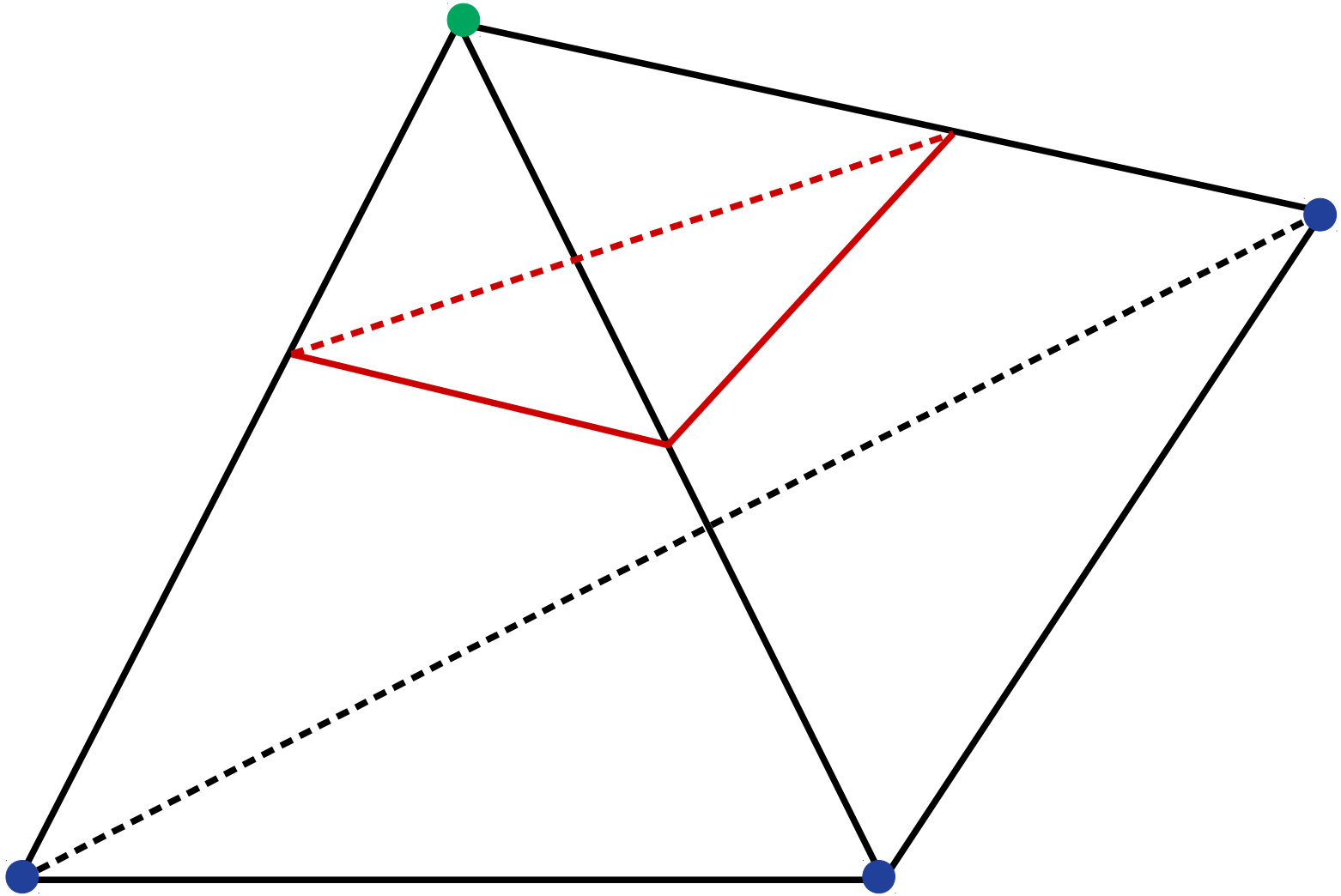
When we now introduce kinked triangle patterns, we can speak about 3D corners and 3D feature lines.

3D

Relevant cases: 3D splits, each split may be kinked or may create a corner

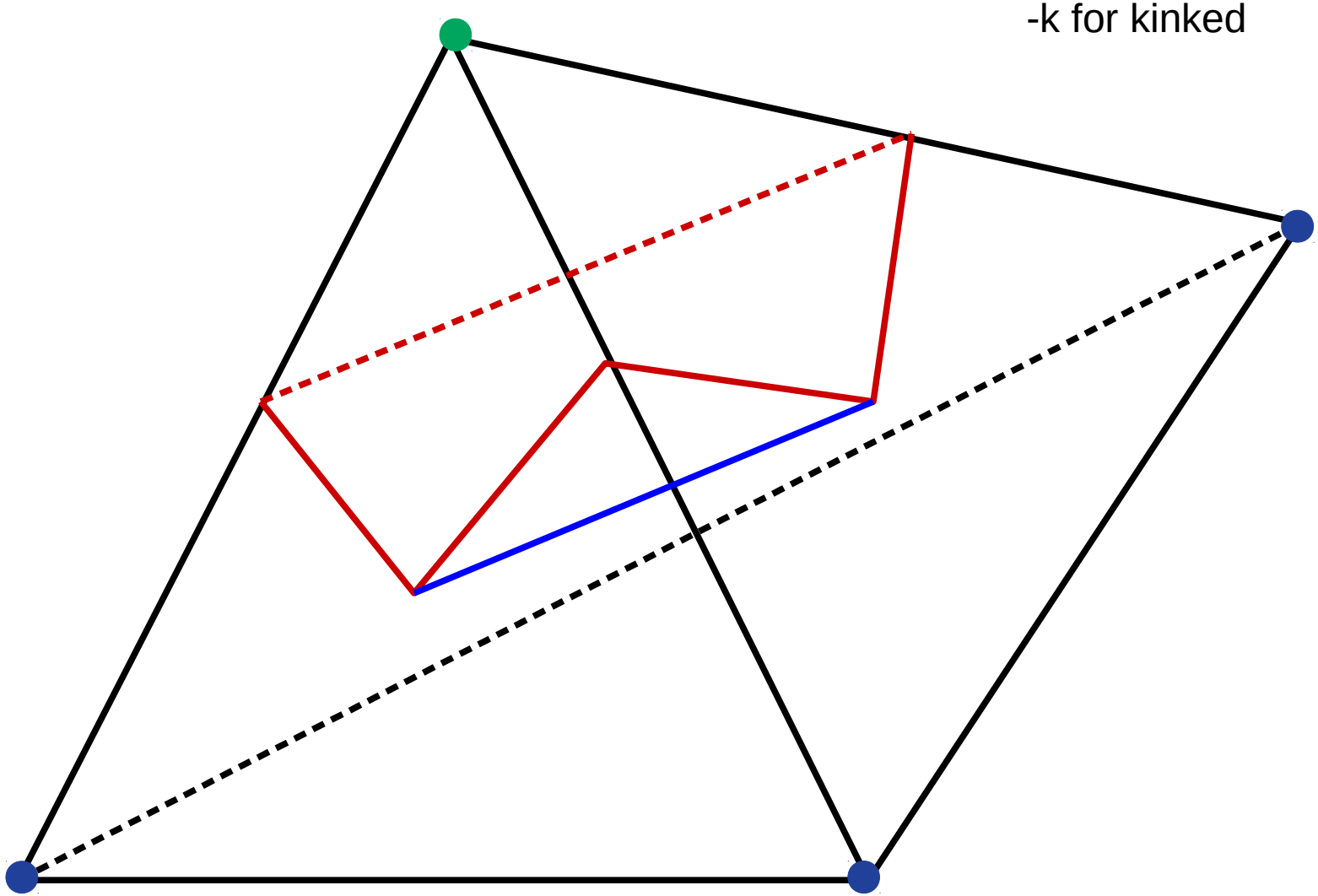
110	110	110	0	1
110	110	110	110	1
200	200	0	0	1
200	200	200	0	2
200	200	200	200	2
2200	200	200	0	1
…				
2220	2110	2110	2110	3

Triangles 110 110 110 000

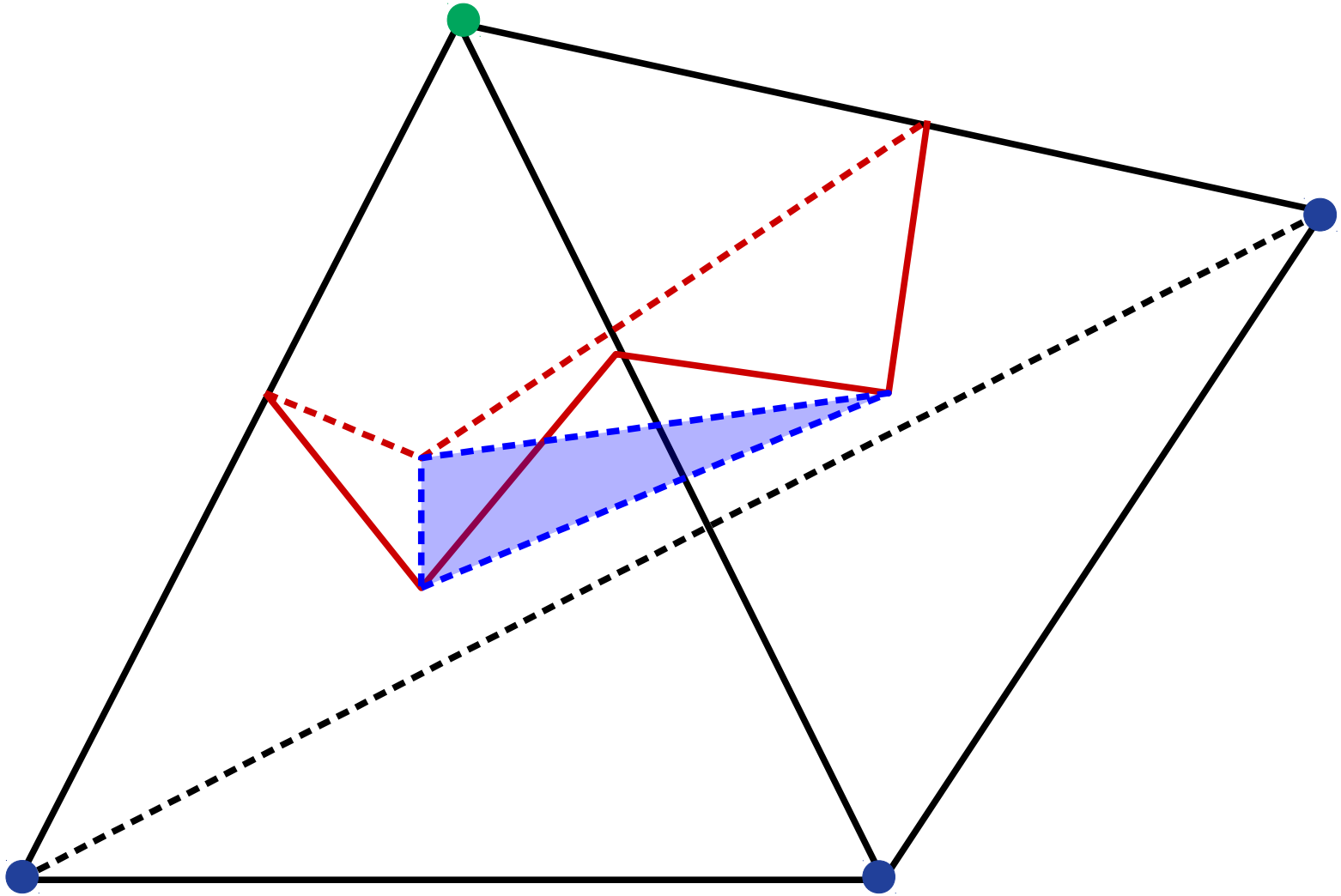


Triangles 110-k 110-k 110 000

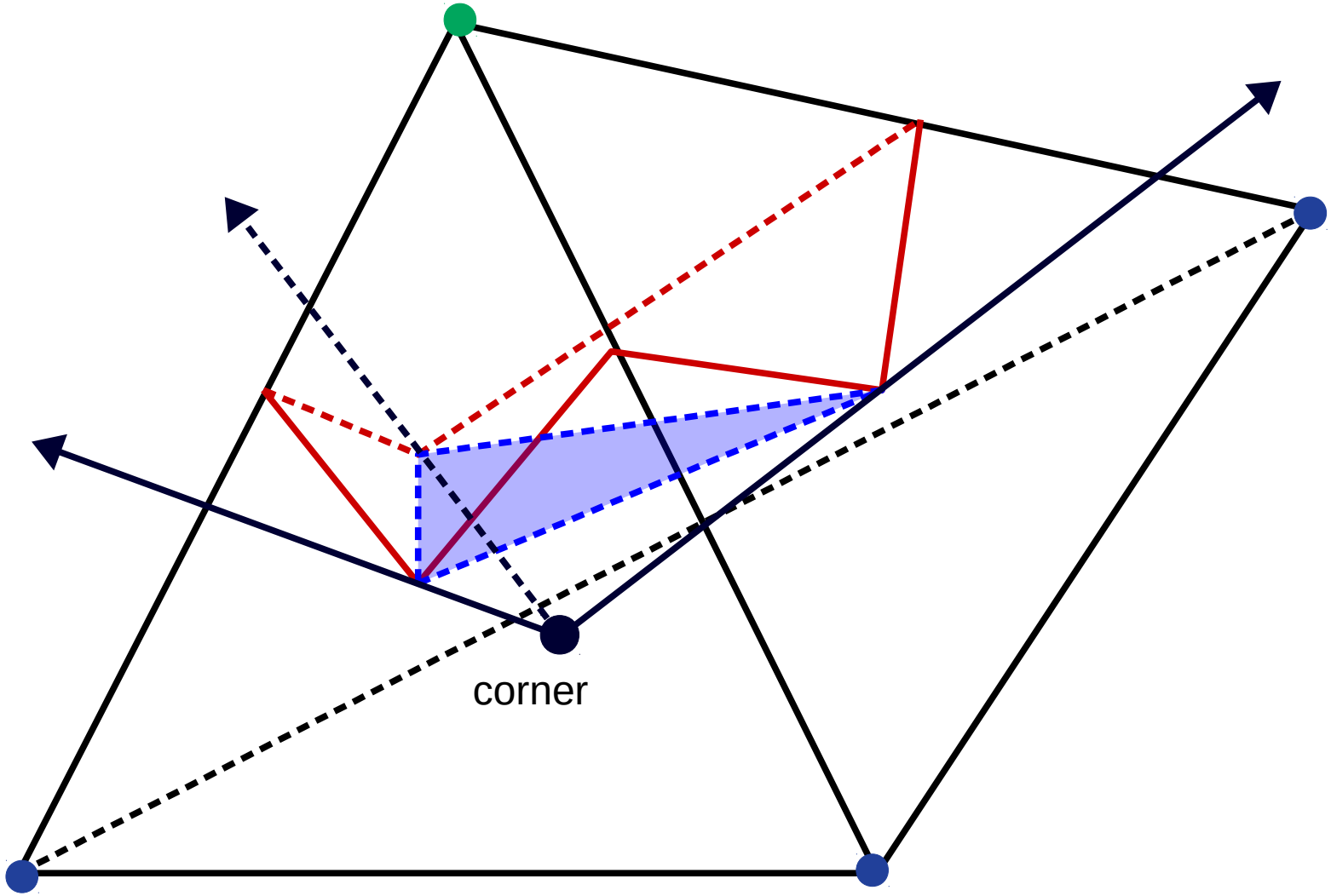
-k for kinked

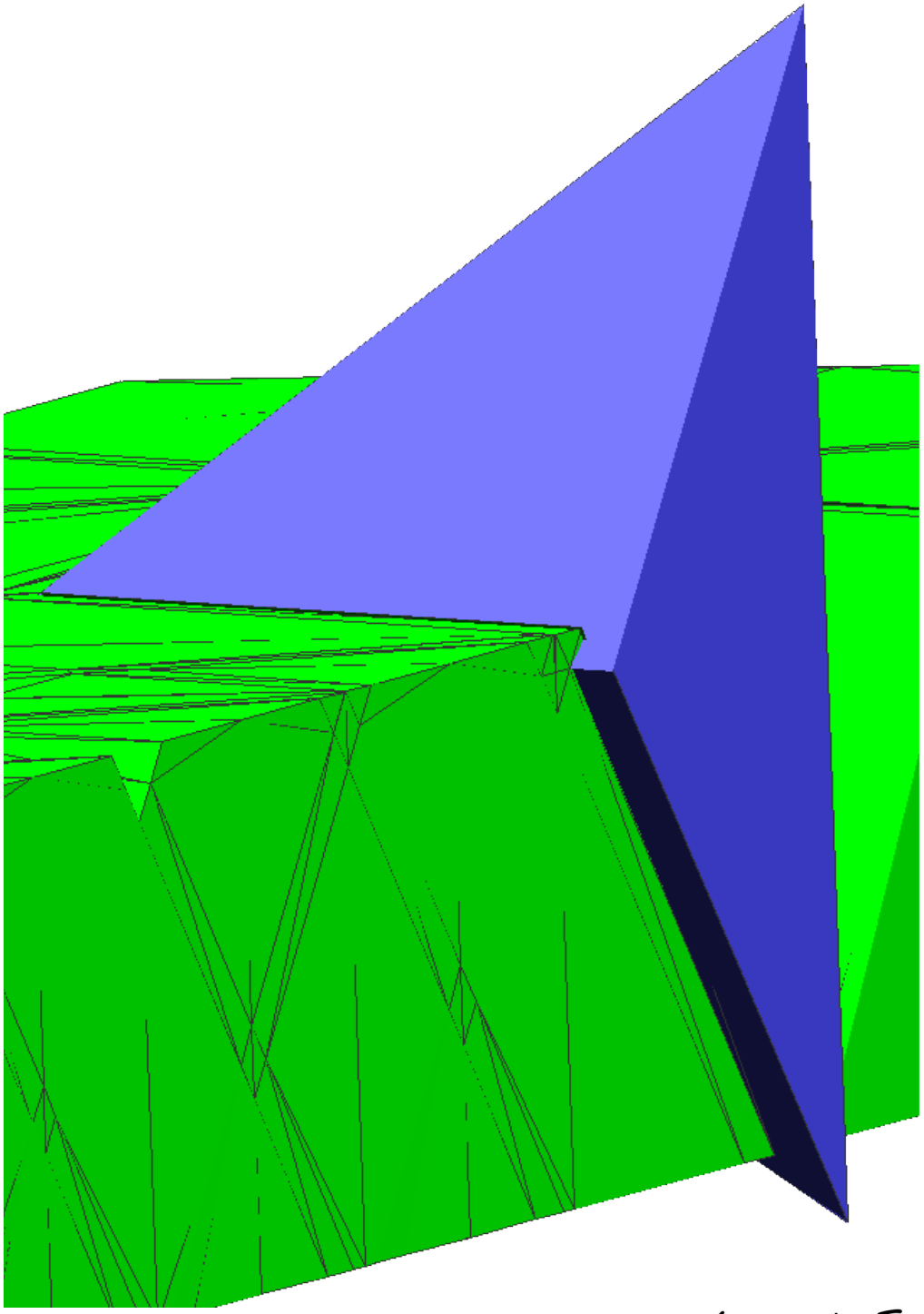


Triangles 110-k 110-k 110-k 000



Triangles 110-k 110-k 110-k 000





110 110 110 0

2 kinked blue faces. Here the kinks should not be connected inside the blue tet.

Situation cannot be solved correctly,
Bisection fails
Tet to 8 Tet refinement is required
in the VMT routine

VMT refines
- edges
- triangles
- tetras

which cannot be solved correctly

edges	2 and more cuts
triangles	kink outside
Tetras	3 or 4 110 faces
	kinks cannot be connected
	2 kinks
	3 or 4 kinks: no corner or
	corner outside

MT element quality

Each node of an edge triangle tetra refinement has a best location.
The barycentric coordinates are:

Edge bisection	0.5 0.5
Edge trisection	1/3 2/3
Triangle split 200	1/3 1/3 1/3
Kinked triangle cuts	0.5 node A + 0.5 node B
Tetra corners	1/3 node A + 1/3 node B + 1/3 node C
or	0.25 node A + .. + 0.25 node D

Bad locations are replaced by

(1.0 - alpha) * bad location + alpha * best location

This strategy damages the geometry but keeps the element quality > 0.0