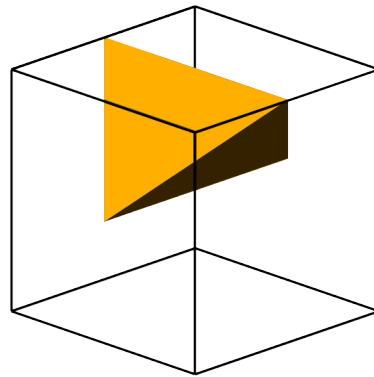


*Lautsch Finite Elemente GmbH*



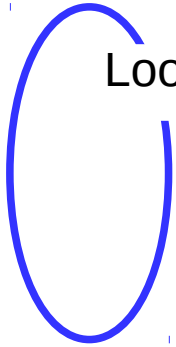
## **Marching Tetras**

1.12.2024

*Lautsch Finite Elemente*

## THE MULTIGRID MARCHING TETRA METHOD

**BCC refinement:** A single cubic Tetra covers all parts



Loop: local refinement: Refine BCC tetras when MT refinement would fail.

Keep element quality  $> 0.155$  by adding edge bisections.

All nodes are BCC.

Final local refinement step:

Each **node** is assigned to exactly one part.

Each **tetra** is assigned to exactly one part.

**MT refinement:** Tetras which are hit by the part geometry are split appropriately.

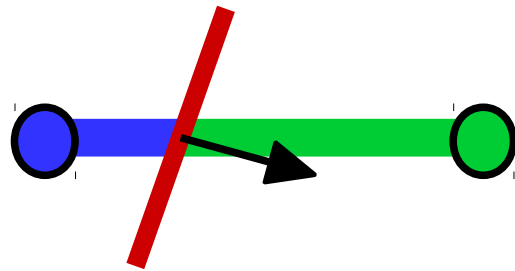
Keep element quality  $>$  user requirement by barycentric limits or other means.

Barycentric limits keep worst element quality  $>$  users requirement.

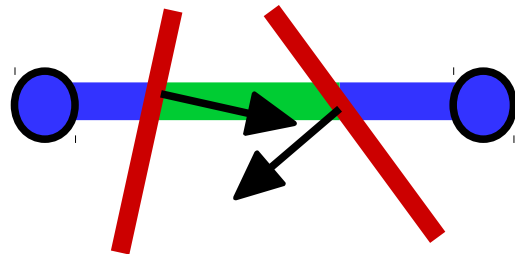
Edge: If an edge bisection is too close to the nodes of the edge, the bisection is moved slightly to the middle of the edge.

Triangle: If a kink-node is created too close to the edges of the tetra, the kink node is moved slightly to the middle of the triangle.

Tetra: If a corner-node is created too close to the triangles of the tetra, the corner node is moved slightly to the middle of the tetra.



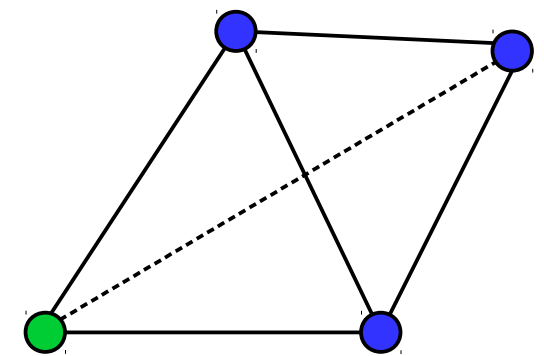
edge - bisection



edge - trisection

The location of the edge cut and the normals are used to predict corners.  
The 1D refinement patterns cause 2D triangle refinement patterns.  
The 2D refinement patterns cause 3D tetra refinement patterns.

We discuss some Marchig Tetra refinement strategies for tetras with different part assignments



	tetra fails	refinement terminates	element quality	geometry quality
1. do nothing	always	never	good	poor
2. mid bisect edges	never	at once	good	ugly
3. bisect edges	> 1 cut per edge	smooth geometry	poor	poor at edged geometry
4. bisect edges + create corners	corner outside	no	poor	
5. bisect edges + create corners + multipart	corner outside	no	poor	it works: disc brake, bicycle, ISS <b>RELEASE 7</b>
6. bisect + trisect edges + create corners + multipart	rarely	2D simple parts		<b>RELEASE 9</b>

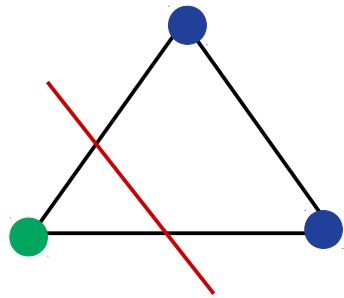
Recursive geometry indicates that you always find parts which cannot be meshed correctly. You cannot prevent MMT from failing locally at the final step.

Either we do nothing. The combined fluid – solid mesh has holes.

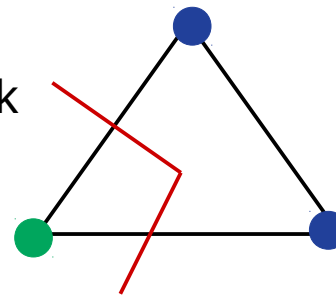
Or we perform stopgap solutions.

Example: If a corner lies out of the tetra  
it is moved to a convenient position inside the tetra.

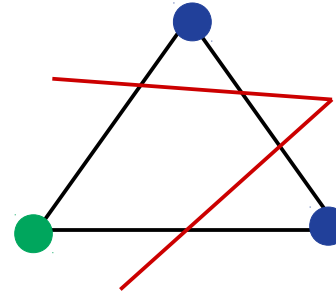
Bisection 110



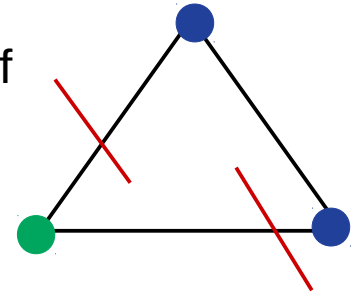
110 + kink



fails, if the kink is outside of the triangle

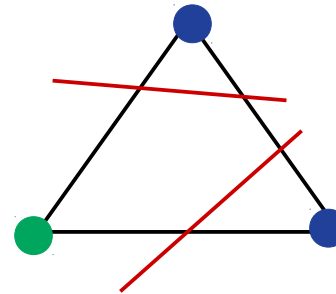


or if

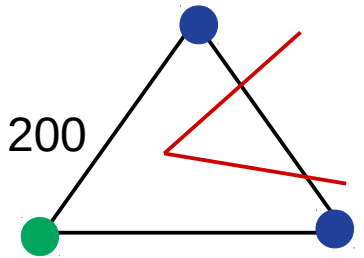


The left failure is healed when we allow edge trisections

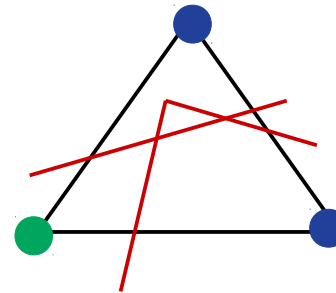
211



200

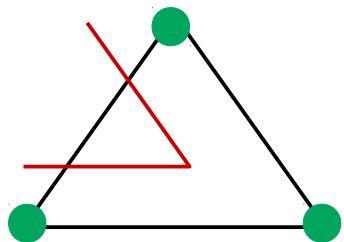


We then have to consider some more refinement patterns, which of course can fail in their own way.



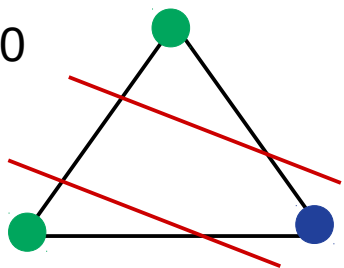
# Trisection

200

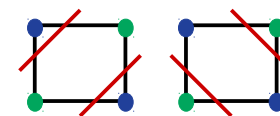
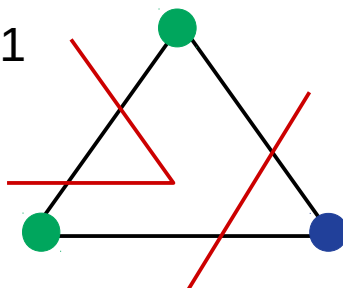


211, ambiguous

211-0



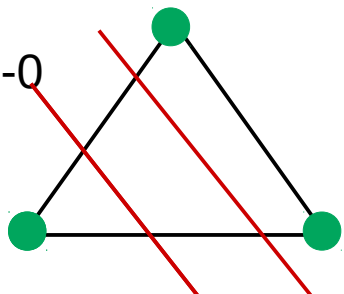
211-1



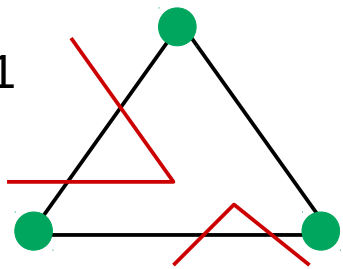
similar to the quad case

220, ambiguous

220-0

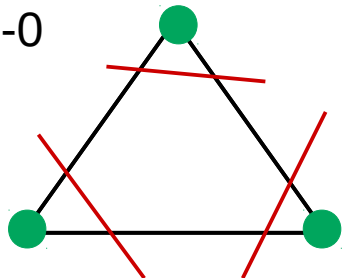


220-1

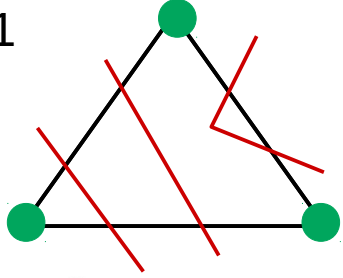


222 3 cases

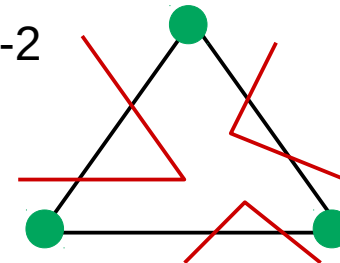
222-0



222-1



222-2





## 3D

Relevant cases:                      3D splits,                      each 3D split may be 2-kinked or 3-kinked ...  
i.e. the number of the triangle kinks

110   110   110   0                      1

110   110   110   110                      1

200   200   0   0                      1

200   200   200   0                      2

200   200   200   200                      2

2200   200   200   0                      1

...

2220   2110   2110   2110                      3

2220   2220   2220   2220                      4

A 3D split fails if an edge or a face or the tetra itself fails.

A 2-kinked 3D split fails if the kinks are not aligned.

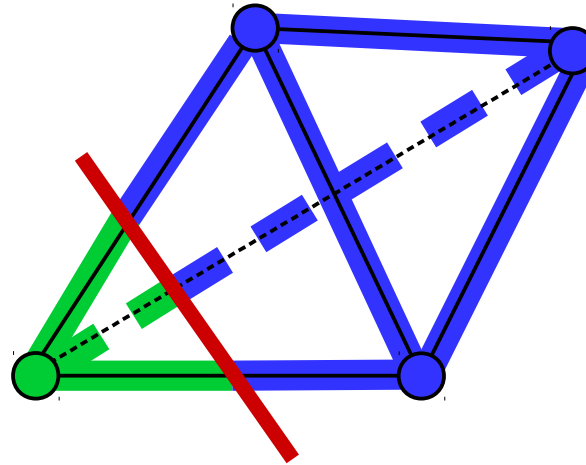
A 3-kinked 3D split may create a corner. It fails when the corner is out of the tetra.

3D

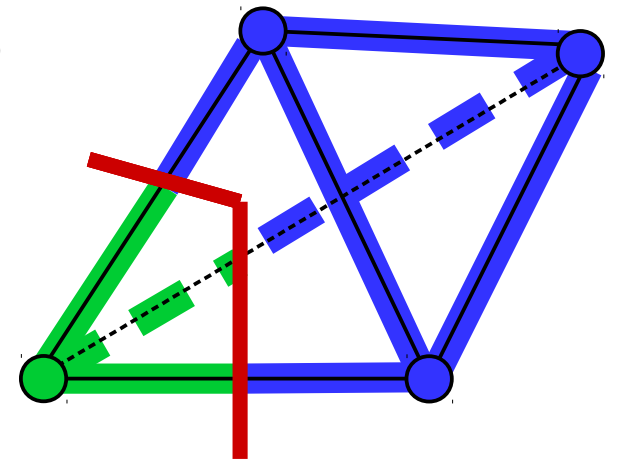
Relevant cases:

Tetra bisection, trisection,

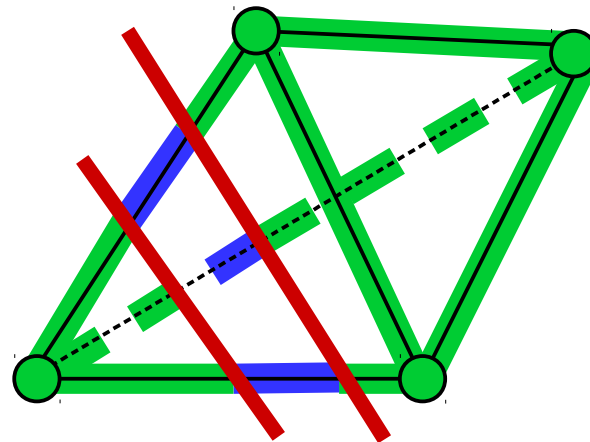
Plane, kinked, corner



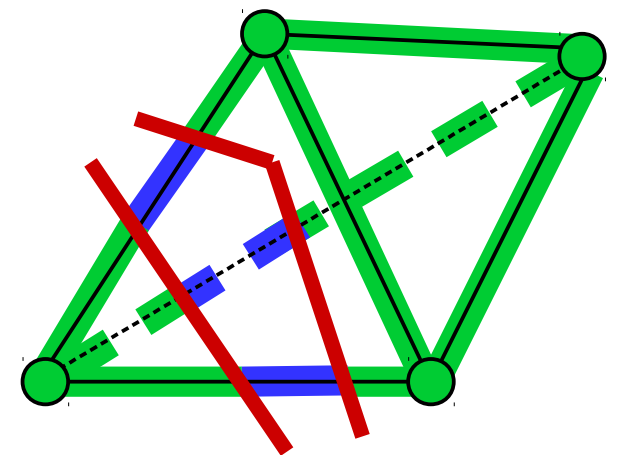
110 110 110 0



110-k 110-k 110 0

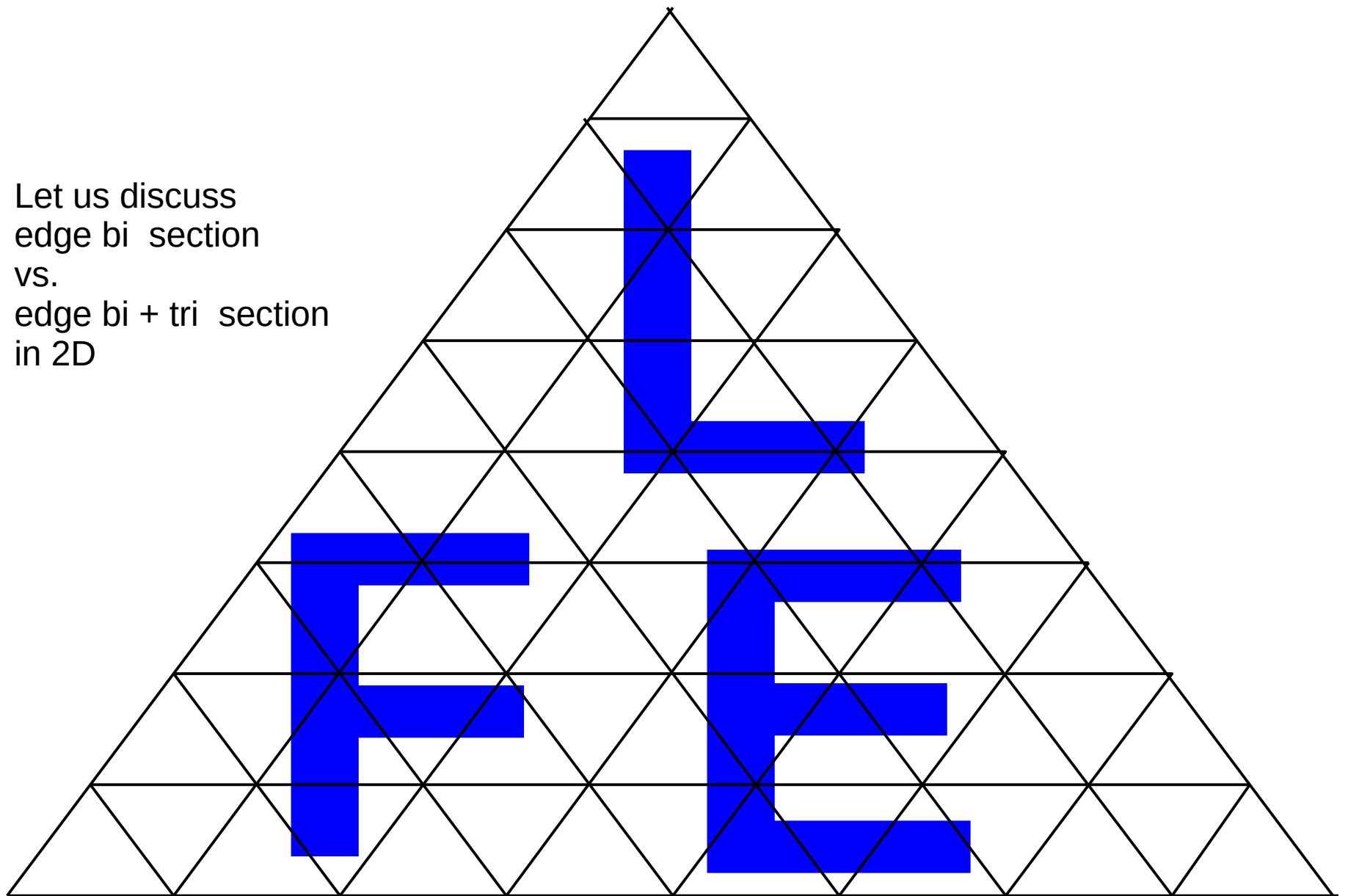


220 220 220 0



220-k 220-k 220 0

Let us discuss  
edge bi section  
vs.  
edge bi + tri section  
in 2D

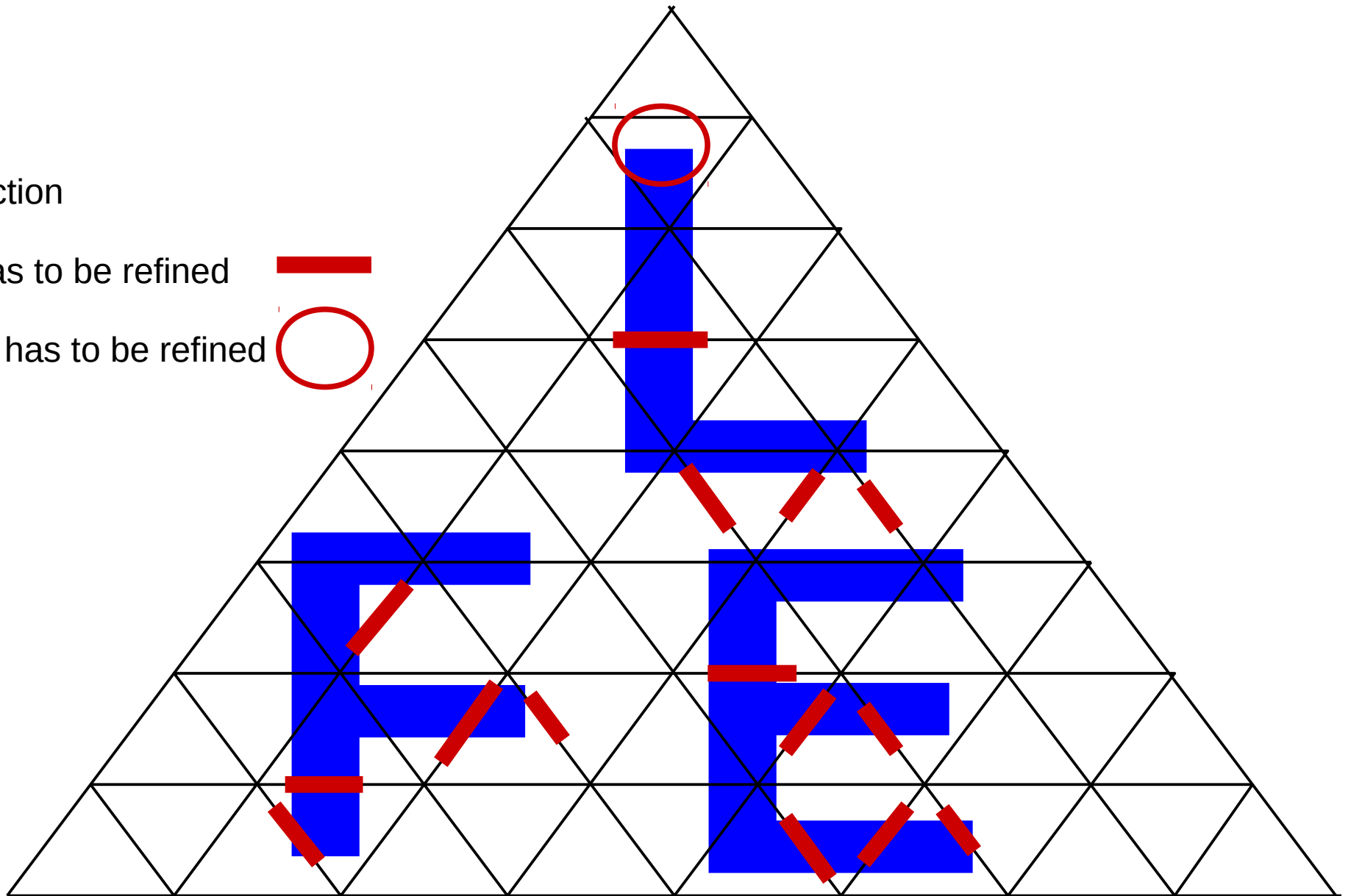
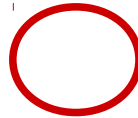


Bi - section

edge has to be refined



triangle has to be refined



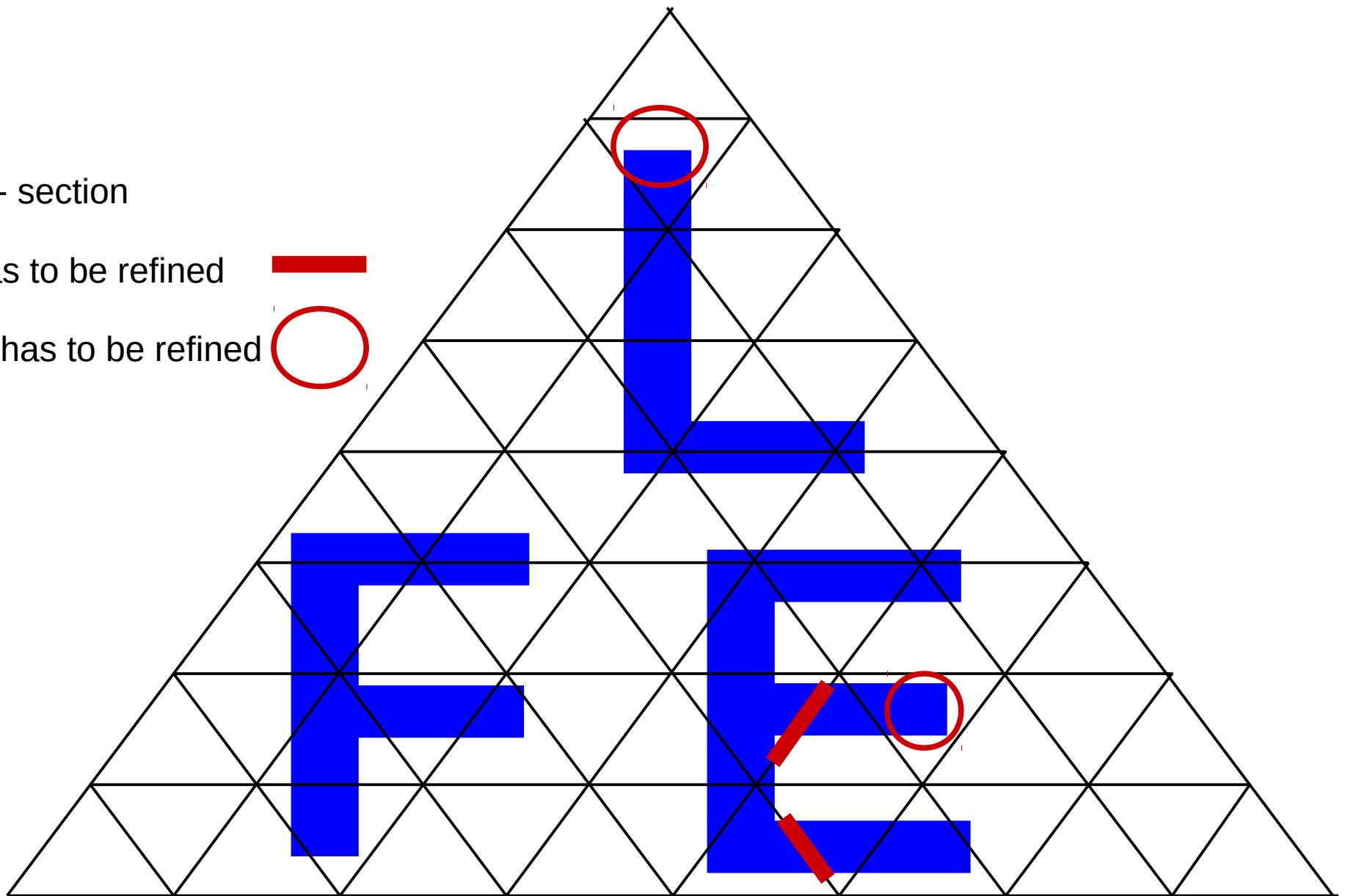
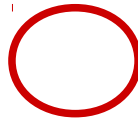
Bi - section

Bi + Tri - section

edge has to be refined



triangle has to be refined



Bi + Tri - section

A rough criterion for 2D bi- and tri- section:

A polygonal bounded part is reconstructed correctly when every triangle does not contain more than one corner.

2D MMT - Bi + Tri – section refinement terminates for simple parts.

The tetra refinement patterns are the logical extension of the triangle refinement patterns on the tetra surface.