Lautsch Finite Elemente GmbH

Bad elements and good results

Adaptive finite elements with anisotropic meshes*

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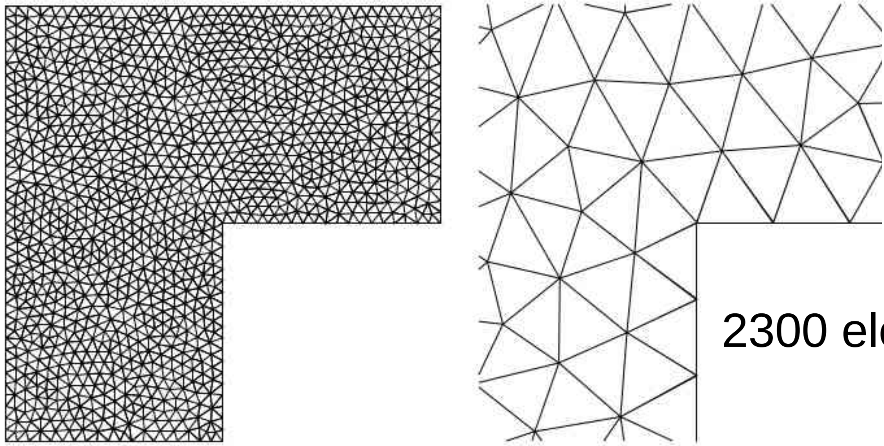
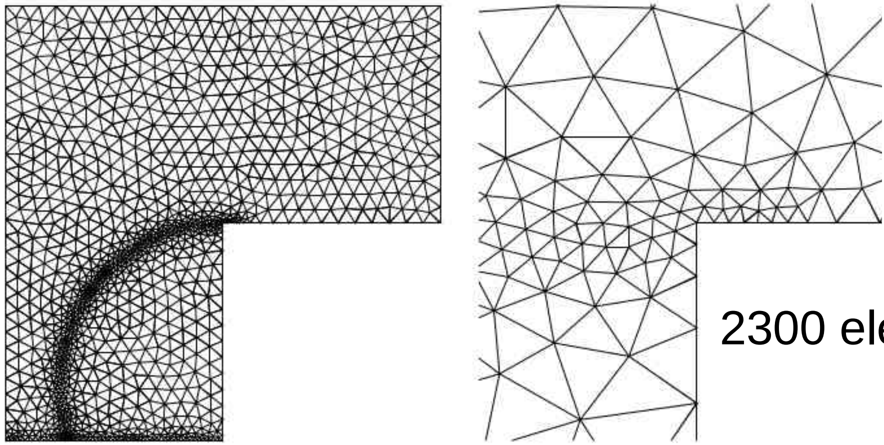


2300 elements Delaunay

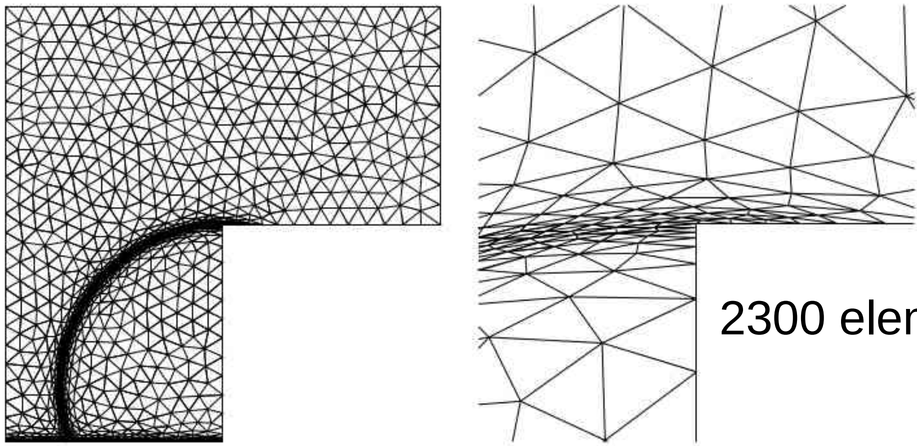
(a) Delaunay: 2326 elements; max. aspect ratio 2.8.

The paper presents a numerical study for the finite element method with anisotropic meshes. We compare the accuracy of the numerical solutions on quasi-uniform, isotropic, and anisotropic meshes for a test problem which combines several difficulties of a corner singularity, a peak, a boundary layer, and a wavefront. Numerical experiment clearly shows the advantage of anisotropic mesh adaptation. The conditioning of the resulting linear equation system is addressed as well. In particular, it is shown that the conditioning with adaptive anisotropic meshes is not as bad as generally assumed.



2300 elements isotropic + adaptive

(b) Isotropic adaptive: 2321 elements; max. aspect ratio 3.0.



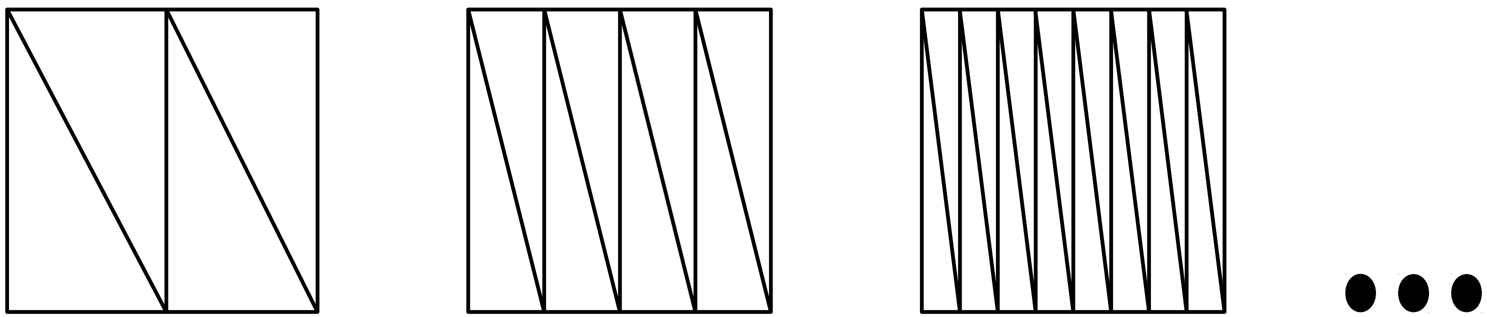
2300 elements anisotropic + adaptive

(c) Anisotropic adaptive: 2316 elements; max. aspect ratio 24.4.

Consider delta u = $u_{xx} + u_{yy} = -2$ at [-1 , 1] x [-1 , 1]

u = 0 at the vertical boundary, du / dy = 0 at the horizontal boundary.

The solution is u(x,y) = 1.0 - x**2



Approximation quality increases and element quality decreases.

Quadratic Elements: The solution is an element of the space to which it should be projected. ANY MESH can represent the solution without discretization errors.