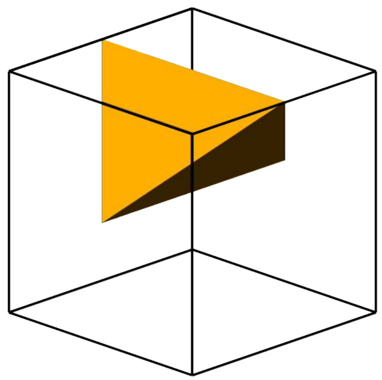
Lautsch Finite Elemente GmbH



Marching Tetras

under construction

28.1.2025

THE MULTIGRID MARCHING TETRA METHOD

… terminates for simple parts.

BCC refinement: A single cubic Tetra covers all parts, its 4 nodes belong to the fluid which is treated like a part.

Loop: local refinement: Refine BCC tetras when MT refinement would fail.

Keep element quality > 0.155 by adding edge bisections.

Final local refinement step:

Each node is assigned to exactly one part.

create **visible nodes**: nodes belong to different parts *

Each tetra is assigned to exactly one part.

MT refinement: Tetras which are hit by the part geometry are split appropriately.

Keep element quality > user requirement by barycentric limits or other means.

* Each Finite Element computation needs a stiffnes matrix, its entries are typically

$$\int u_i D^T S \ D \ u_j$$

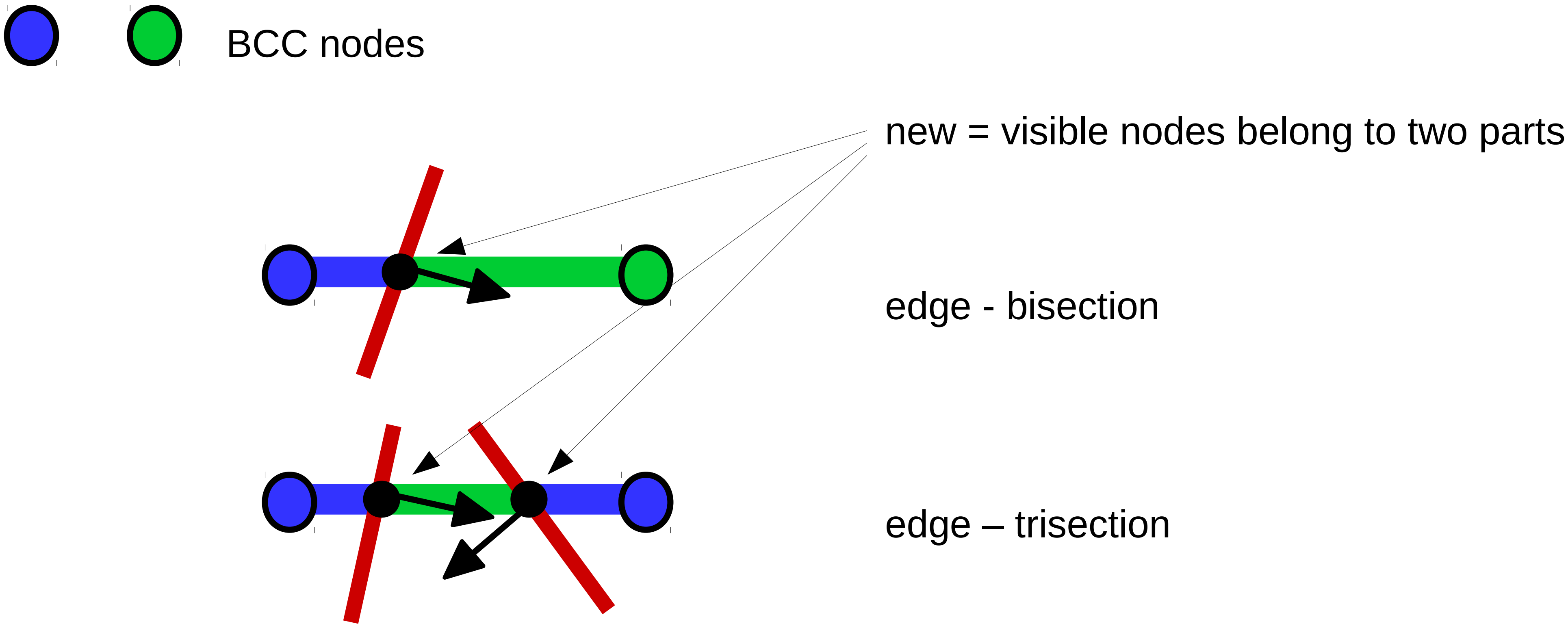
u is a basic function of the Finite Element vector space
D is a differential operator
S is a matrix of physics, defined on Finite Elements. The physics cannot be changed inside the finite element. Physics are a property of the part.

Each part has one and only one physics.
Different parts may have the same physics.

CAD geometry = STL data is well known from rapid prototyping and fits very well to the Finite Element data structure.

How does MMT interact with CAD/STL data ?

Node	belong to one and only one part in the BCC time. If there are several possibilities (overlapping parts), we need rules to decide which part wins. MT (visible) nodes lie on the surface of the part, and e.g. on the surface of the fluid.
Edge	is cut by CAD/STL triangles
Triangle	no direct interaction, but indirect via its edges
Tetra	no direct interaction, but indirect via its edges and triangles

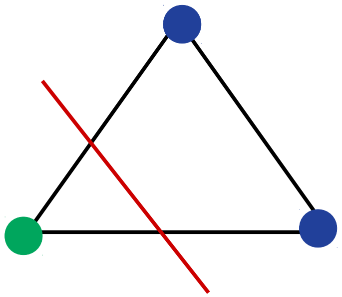


The location of the edge cut and the normals are used to predict corners.

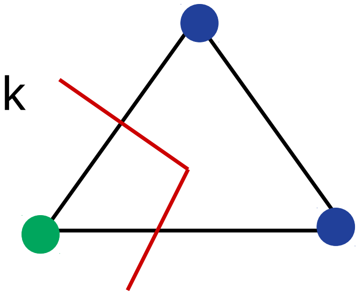
The 1D refinement patterns cause 2D triangle refinement patterns.

The 2D refinement patterns cause 3D tetra refinement patterns.

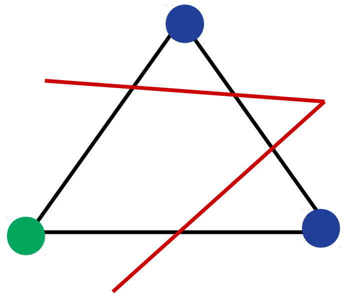
Bisection 110



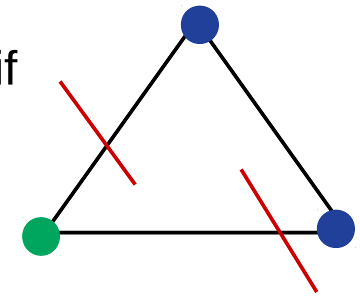
110 + kink



fails, if the kink is outside of the triangle

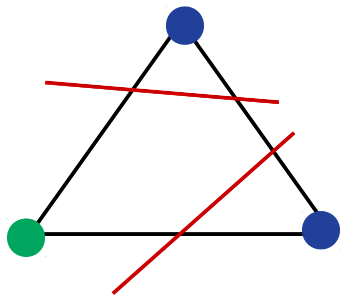


or if

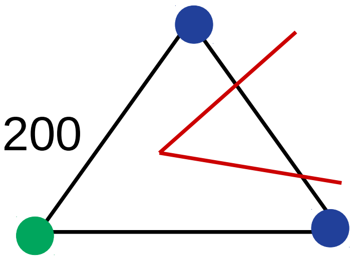


The left failure is healed when we allow edge trisections

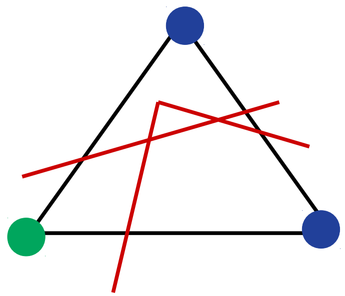
211



200

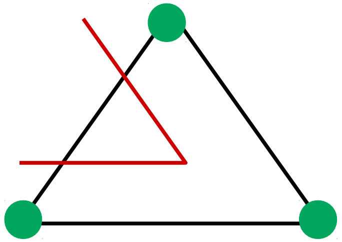


We then have to consider some more refinement patterns,
which of course can fail in their own way.

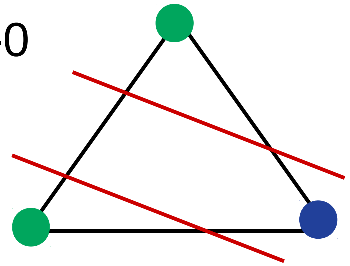


Trisection

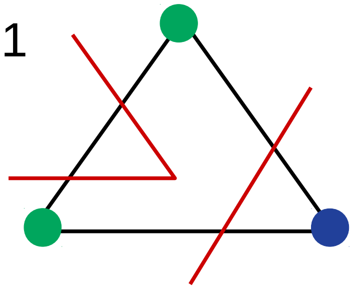
200

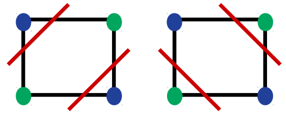


211, ambiguous 211-0



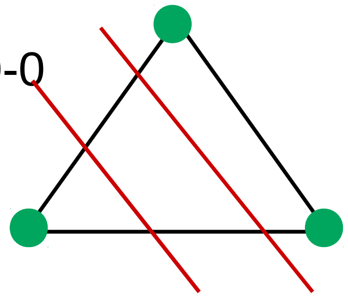
211-1



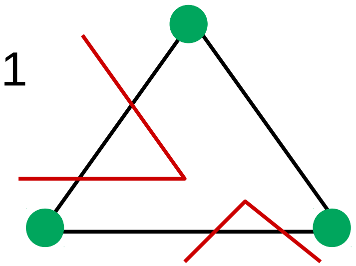


similar to the quad case

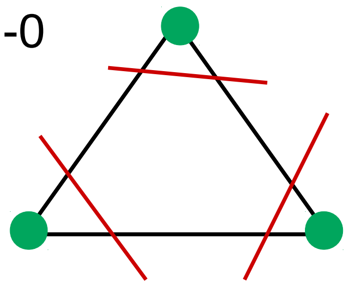
220, ambiguous 220-0



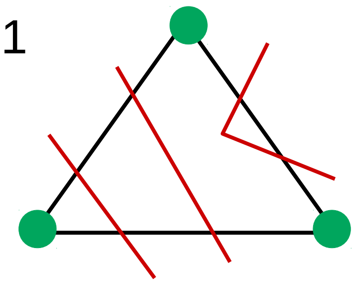
220-1



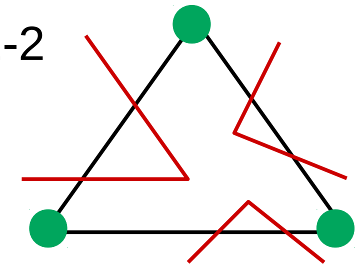
222 3 cases 222-0

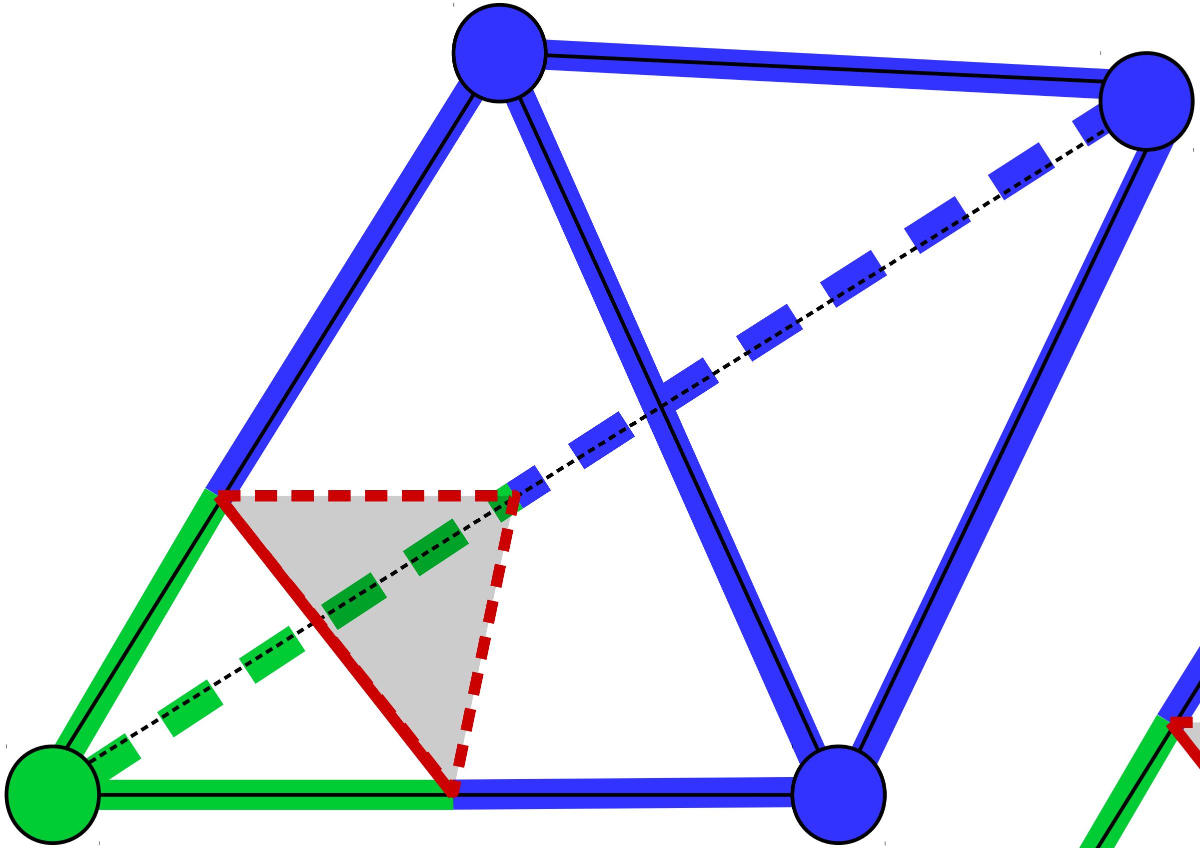


222-1

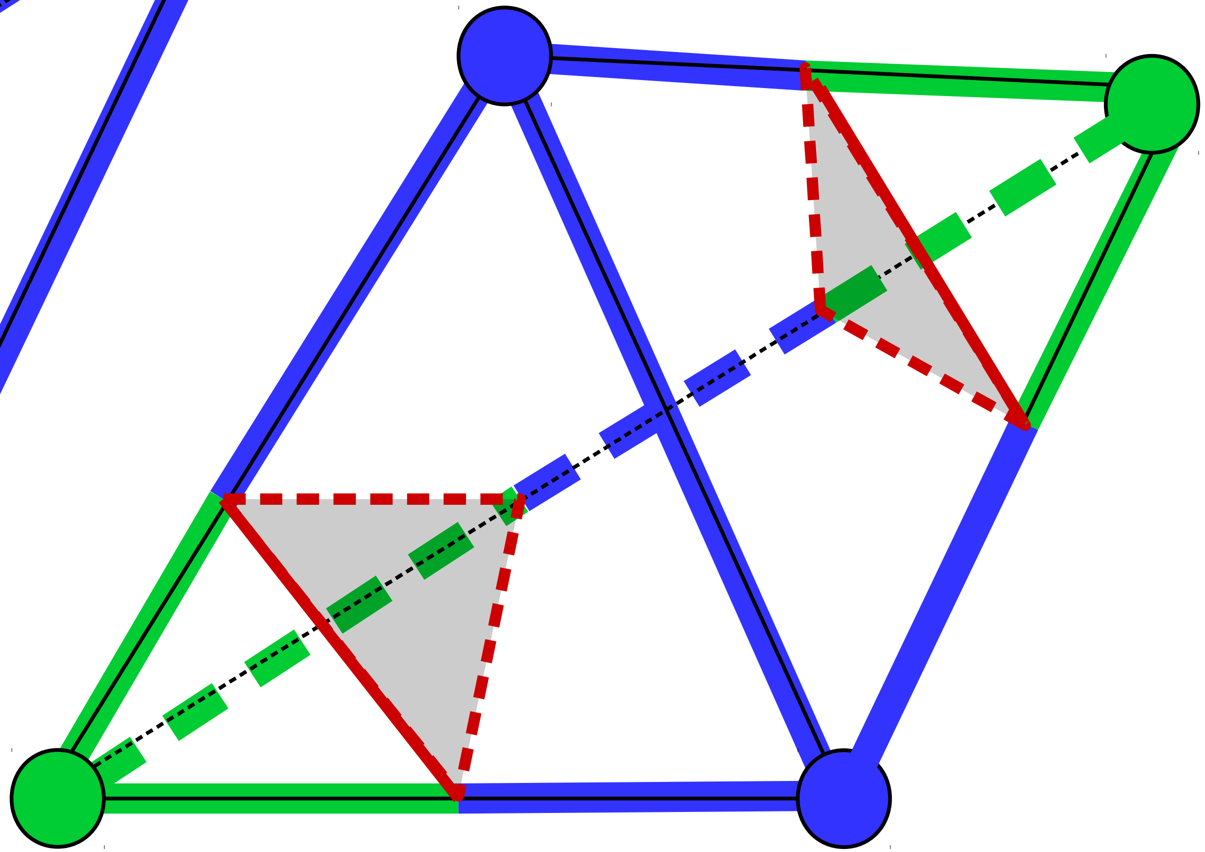


222-2

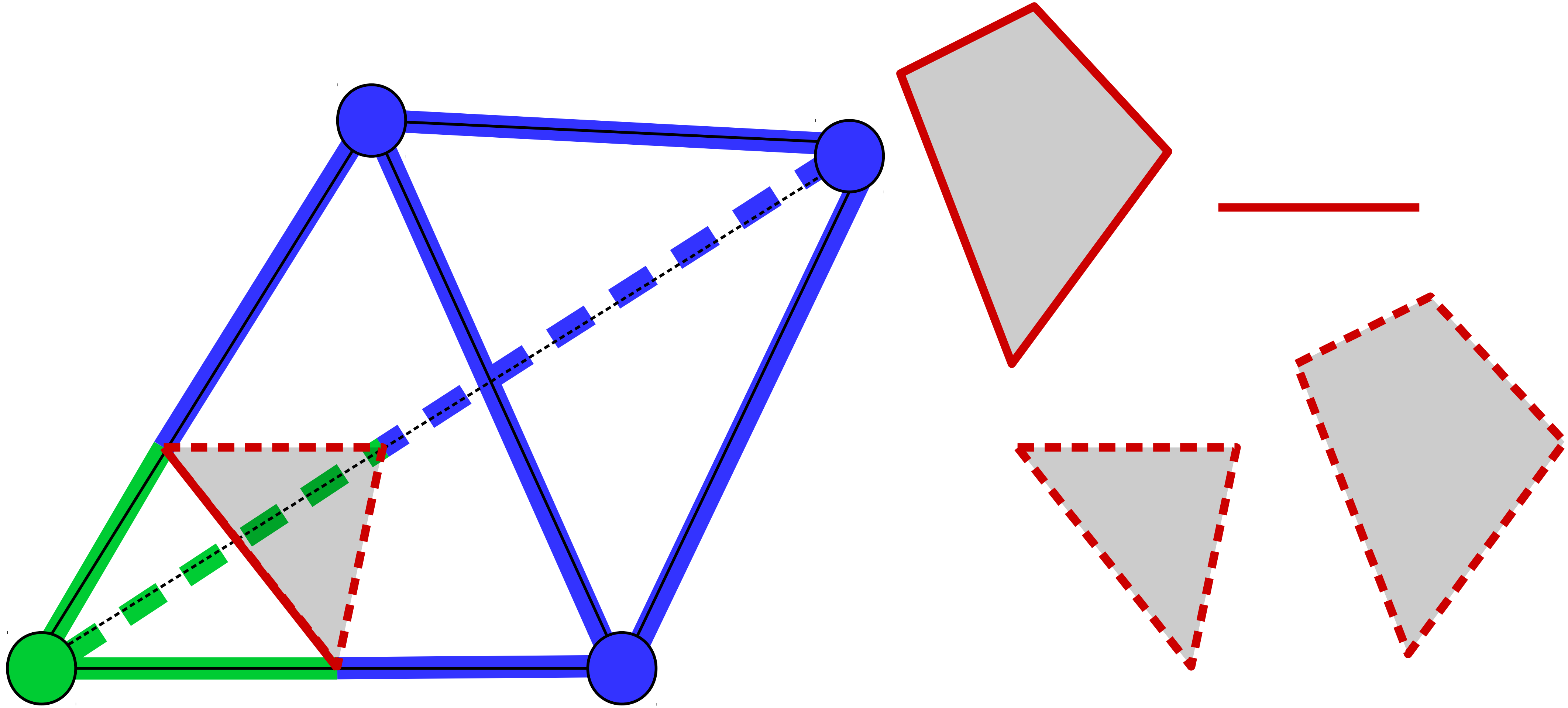




3 110 triangle splits define a tetra bisection



2 211 and 2 110 triangle splits define a tetra trisection



3 110 triangle splits define a tetra bisection

Construction……………………..

3D

Relevant cases: 3D splits, each 3D split may be 2-kinked or 3-kinked …

110	110	110	0	1
110	110	110	110	1
200	200	0	0	1
200	200	200	0	2
200	200	200	200	2
211	211	110	110	2

…

2220	2110	2110	2110	3
2220	2220	2220	2220	4

Only the most important splits are executed. The tetras whose splits are not executed are refined.

3D

edge fails → triangle fails

triangle fails → tetra fails

Reduce the number of cases by intentional failure

triangles	222-1, 222-2, kinked cases …
tetras	...

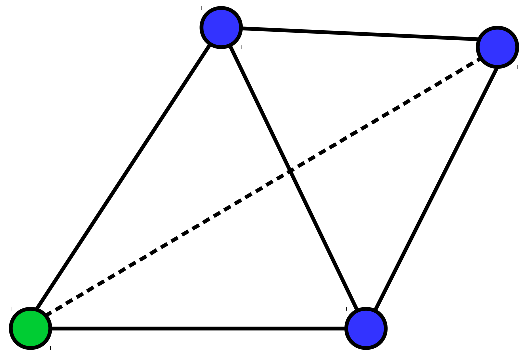
A 2-kinked 3D split fails if the kinks are not aligned.

A 3-kinked 3D split may create a corner (kink point). It fails when the corner is out of the tetra and its neighbours.

Currently only one kink line per tetra.
Currently only one kink point per tetra.

If edge trisection is considered as a failure, release 10 switches to release 7

We discuss some Marchig Tetra refinement strategies
for tetras with geometry contact



edge strategy	tetra fails	refinement terminates	element quality	geometry quality
1. do nothing	always	never	good	poor
2. bisect edges	> 1 cut per edge	smooth geometry	poor	poor at edged geometry
3. bisect edges + create corners	corner outside	no	poor	
4. bisect edges + create corners + multipart	corner outside	no	poor	it works: disc brake, bicycle, ISS **RELEASE 7**
5. bisect + trisect edges + create corners	rarely	simple parts	poor	**RELEASE 9**

RELEASE 9 + multipart = **RELEASE 10**

Does MMT terminate?

MMT terminates, when there are no edges, triangles, tetras which fail at the MT refinement.

Recursive geometry indicates that you always find parts which cannot be meshed Correctly.

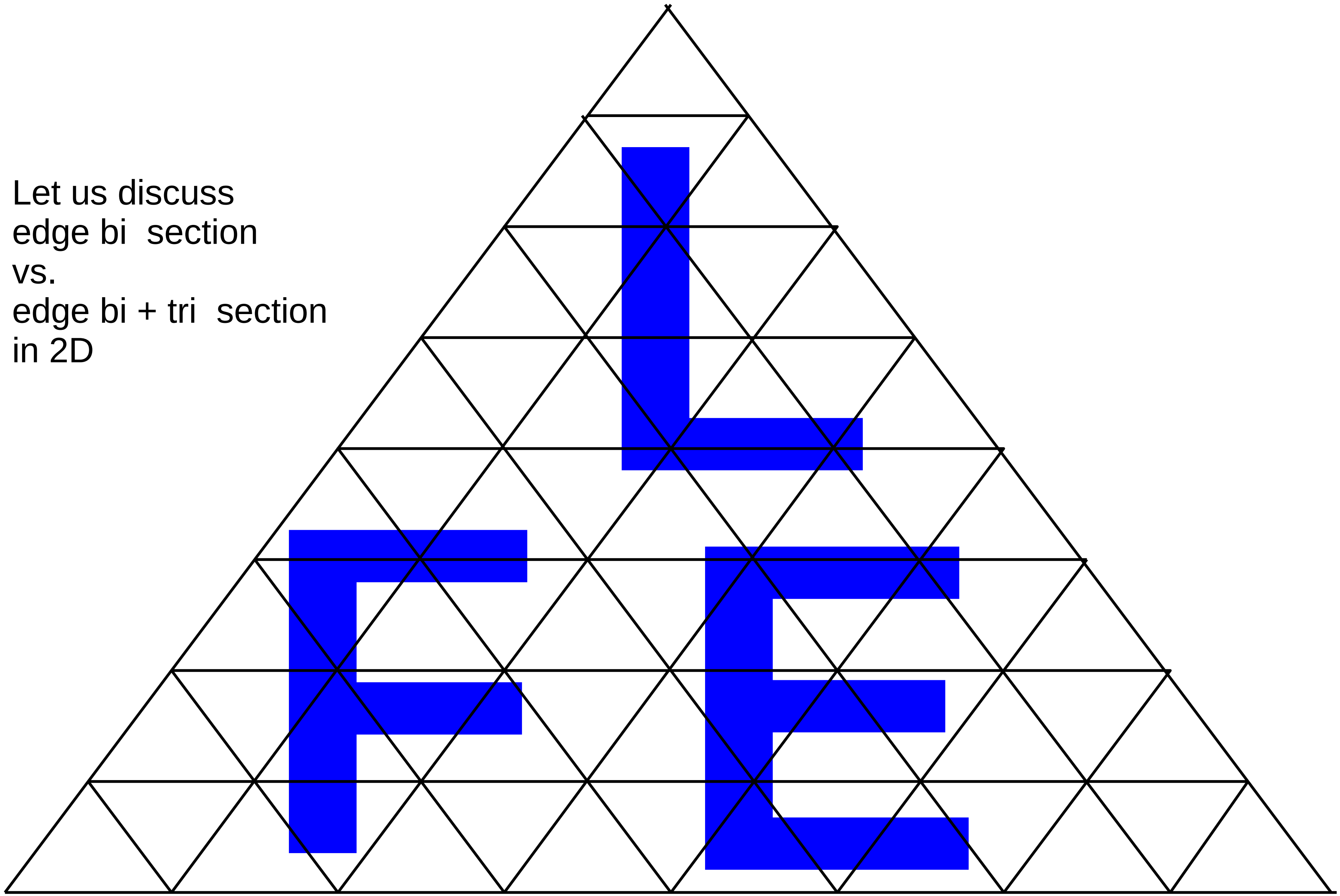
MMT terminates for „simple“ parts.

For „complicated“ parts there are tetras which cannot be refined according to the list of refinement patterns. Either we do nothing. The combined fluid – solid mesh has holes.

Or we perform stopgap solutions.

Example: If a corner lies out of the tetra
it is moved to a convenient position inside the tetra.

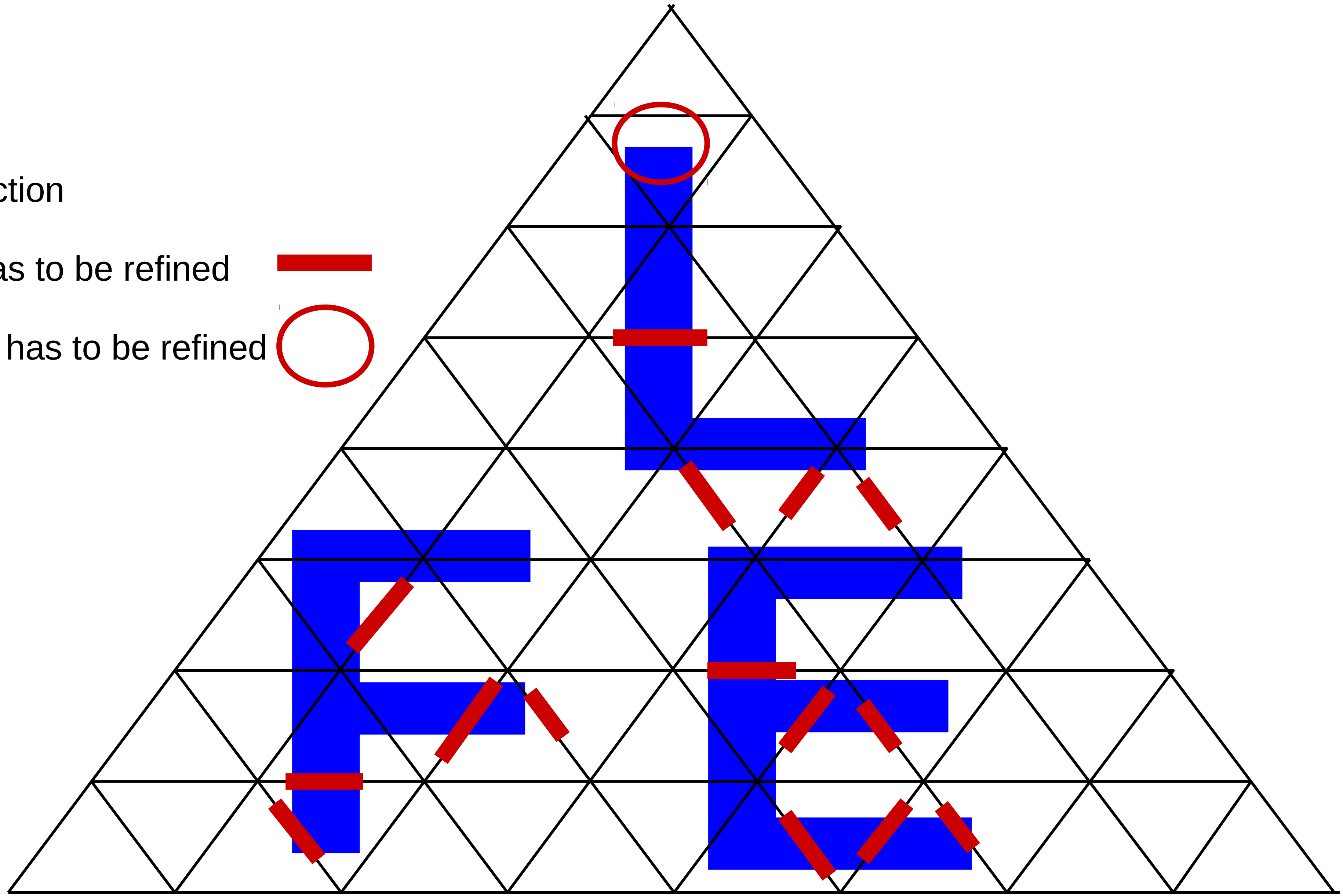
Let us discuss
edge bi section
vs.
edge bi + tri section
in 2D



Bi - section

edge has to be refined

triangle has to be refined

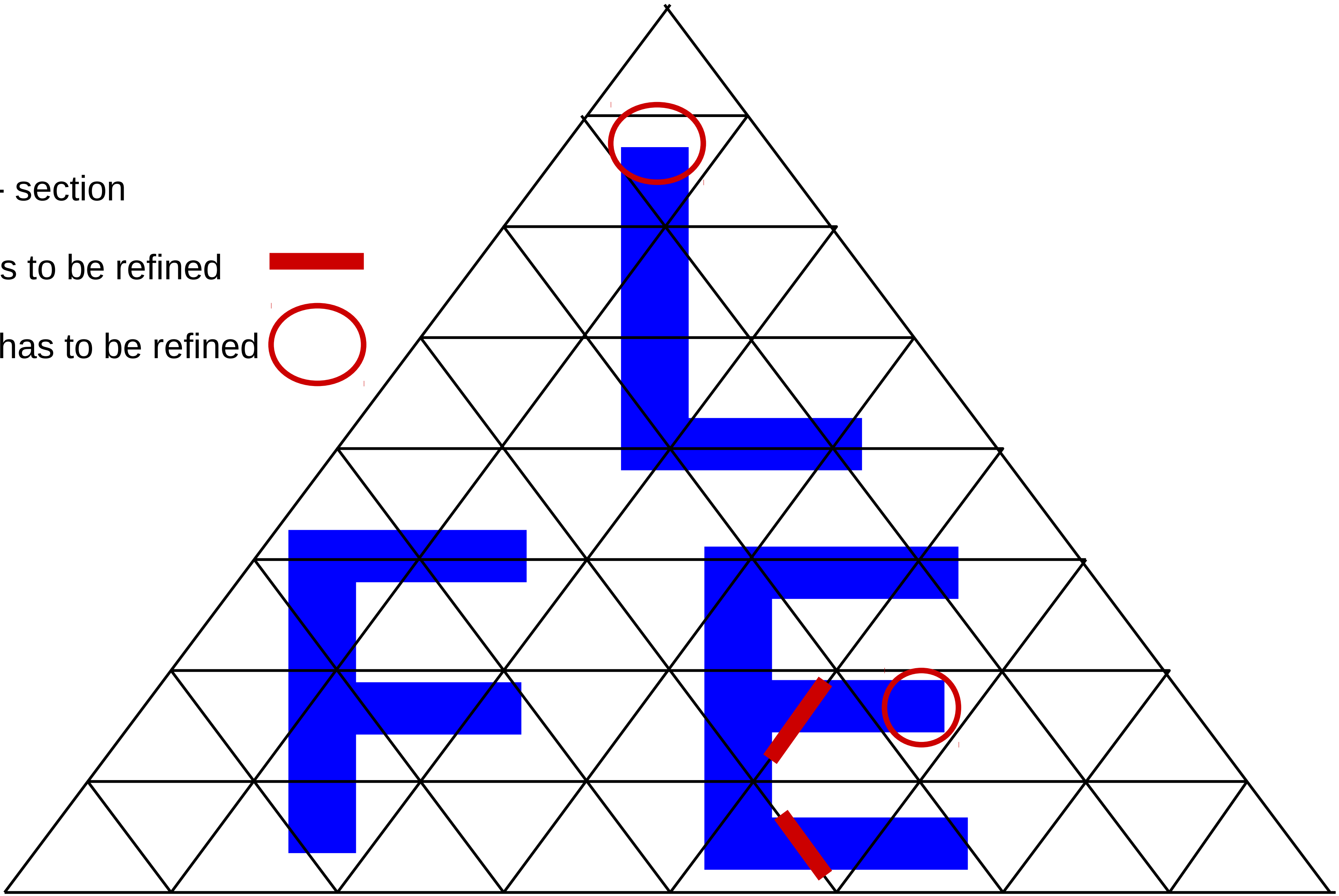


Bi - section

Bi + Tri - section

edge has to be refined

triangle has to be refined



Bi + Tri - section

2D + 3D MMT driven by edge - Bi + Tri – section refinement terminates for simple parts.

Even in 2D there are complicated corners which cannot be reconstructed correctly.

But e.g. a geometric multipart corner cannot be approximated correctly.

Barycentric limits keep worst element quality > user requirement.

Edge: If an edge bisection is too close to the nodes of the edge, the bisection is moved slightly to the middle of the edge.

Triangle: If a kink-node is created too close to the edges of the tetra, the kink node is moved slightly to the middle of the triangle.

Tetra: If a corner-node is created too close to the triangles of the tetra, the corner node is moved slightly to the middle of the tetra.

3 fold 110 k generate a corner